Yildiz Technical University

Department of Mechanical Engineering

Machine Theory, System Dynamics and Control Division

Special Laboratory - Position Control of Rotary Servo Base Unit using PIV Controller

Lab Date:

Number:

Name Surname:

Lab Director:

Group/Sub-group: ….. / ….

Lab Location: O Block - Automatic Control Laboratory
Lab Name: Machine Theory - 3
Subject: Position Control of Rotary Servo Base Unit (SRV02) using PV and PIV Controllers

Apparatus and tools:

- Computer with MATLAB-Simulink and QUARC software

- Data acquisition device, power amplifier, and main components of the SRV02 (e.g. actuator, sensors).

Aim of the experiment:

-Deriving the dynamics equation and transfer function for the SRV02 servo plant using the first-principles.

- Design of a proportional-velocity (PV) controller for position control of the servo load shaft to meet certain time-domain requirements.

-Design of a proportional-velocity-integral (PIV) controller to track a ramp reference signal.

-Implementation of the controllers on the Quanser SRV02 device to evaluate their performance.

Modeling

The objective of this experiment is to find a transfer function that describes the rotary motion of the SRV02 load shaft. The dynamic model is derived analytically from classical mechanics principles and using experimental methods.

Topics Covered

- Deriving the dynamics equation and transfer function for the SRV02 servo plant using the first-principles.
- Obtaining the SRV02 transfer function using a frequency response experiment.
- Obtaining the SRV02 transfer function using a bump test.

- Tuning the obtained transfer function and validating it with the actual system response.

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- Data acquisition device (e.g. Q2-USB), the power amplifier (e.g. VoltPAQ-X1), and the main components of the SRV02 (e.g. actuator, sensors), as described in References [2], [4], and [6], respectively.
- Wiring and operating procedure of the SRV02 plant with the amplifier and data-aquisition (DAQ) device, as discussed in Reference [6].
- Transfer function fundamentals, e.g. obtaining a transfer function from a differential equation.
- Laboratory described in Appendix A to get familiar with using QUARC[r]with the SRV02.

1.1 Background

The angular speed of the SRV02 load shaft with respect to the input motor voltage can be described by the following first-order transfer function

$$\frac{\Omega_l(s)}{V_m(s)} = \frac{K}{(\tau s + 1)} \tag{1.1.1}$$

where $\Omega_l(s)$ is the Laplace transform of the load shaft speed $\omega_l(t)$, $V_m(s)$ is the Laplace transform of motor input voltage $v_m(t)$, K is the steady-state gain, τ is the time constant, and s is the Laplace operator.

The SRV02 transfer function model is derived analytically in Section 1.1.1 and its K and τ parameters are evaluated. These are known as the nominal model parameter values. The model parameters can also be found experimentally. Sections 1.1.2.1 and 1.1.2.2 describe how to use the frequency response and bump-test methods to find K and τ. These methods are useful when the dynamics of a system are not known, for example in a more complex system. After the lab experiments, the experimental model parameters are compared with the nominal values.

1.1.1 Modeling Using First-Principles

1.1.1.1 Electrical Equations

The DC motor armature circuit schematic and gear train is illustrated in Figure 1.1. As specified in [6], recall that R_m is the motor resistance, L_m is the inductance, and k_m is the back-emf constant.

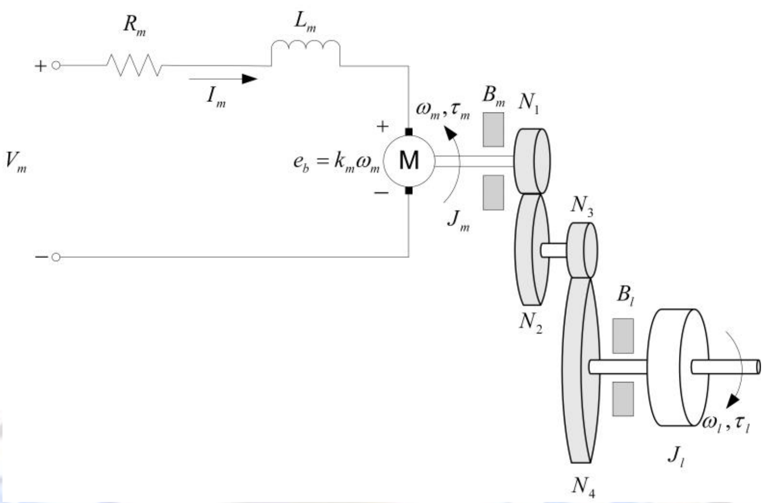

Figure 1.1: SRV02 DC motor armature circuit and gear train

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, ω_m, and the back-emf constant of the motor, k_m. It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t) \tag{1.1.2}$$

Using Kirchoff's Voltage Law, we can write the following equation:

$$V_m(t) - R_m I_m(t) - L_m \frac{dI_m(t)}{dt} - k_m \omega_m(t) = 0 \tag{1.1.3}$$

Since the motor inductance L_m is much less than its resistance, it can be ignored. Then, the equation becomes

$$V_m(t) - R_m I_m(t) - k_m \omega_m(t) = 0 \tag{1.1.4}$$

Solving for $I_m(t)$, the motor current can be found as:

$$I_m(t) = \frac{V_m(t) - k_m \omega_m(t)}{R_m} \tag{1.1.5}$$

1.1.1.2 Mechanical Equations

In this section the equation of motion describing the speed of the load shaft, ω_l, with respect to the applied motor torque, τ_m, is developed.

Since the SRV02 is a one degree-of-freedom rotary system, Newton's Second Law of Motion can be written as:

$$J \cdot \alpha = \tau \tag{1.1.6}$$

where J is the moment of inertia of the body (about its center of mass), α is the angular acceleration of the system, and τ is the sum of the torques being applied to the body. As illustrated in Figure 1.1, the SRV02 gear train along with the viscous friction acting on the motor shaft, B_m, and the load shaft B_l are considered. The load equation of motion is

$$J_l \frac{d\omega_l(t)}{dt} + B_l \omega_l(t) = \tau_l(t) \quad (1.1.7)$$

where J_l is the moment of inertia of the load and τ_l is the total torque applied on the load. The load inertia includes the inertia from the gear train and from any external loads attached, e.g. disc or bar. The motor shaft equation is expressed as:

$$J_m \frac{d\omega_m(t)}{dt} + B_m \omega_m(t) + \tau_{ml}(t) = \tau_m(t) \quad (1.1.8)$$

where J_m is the motor shaft moment of inertia and τ_{ml} is the resulting torque acting on the motor shaft from the load torque. The torque at the load shaft from an applied motor torque can be written as:

$$\tau_l(t) = \eta_g K_g \tau_{ml}(t) \quad (1.1.9)$$

where K_g is the gear ratio and η_g is the gearbox efficiency. The planetary gearbox that is directly mounted on the
SRV02 motor (see [6] for more details) is represented by the N_1 and N_2 gears in Figure 1.1 and has a gear ratio of

$$K_{gi} = \frac{N_2}{N_1} \quad (1.1.10)$$

This is the *internal* gear box ratio. The motor gear N_3 and the load gear N_4 are directly meshed together and are visible from the outside. These gears comprise the *external* gear box which has an associated gear ratio of

$$K_{ge} = \frac{N_4}{N_3} \quad (1.1.11)$$

The gear ratio of the SRV02 gear train is then given by:

$$K_g = K_{ge} K_{gi} \quad (1.1.12)$$

Thus, the torque seen at the motor shaft through the gears can be expressed as:

$$\tau_{ml}(t) = \frac{\tau_l(t)}{\eta_g K_g} \quad (1.1.13)$$

Intuitively, the motor shaft must rotate K_g times for the output shaft to rotate one revolution.

$$\theta_m(t) = K_g \theta_l(t) \quad (1.1.14)$$

We can find the relationship between the angular speed of the motor shaft, ω_m, and the angular speed of the load shaft, ω_l by taking the time derivative:

$$\omega_m(t) = K_g \omega_l(t) \quad (1.1.15)$$

To find the differential equation that describes the motion of the load shaft with respect to an applied motor torque substitute (1.1.13), (1.1.15) and (1.1.7) into (1.1.8) to get the following:

$$J_m K_g \frac{d\omega_l(t)}{dt} + B_m K_g \omega_l(t) + \frac{J_l(\frac{d\omega_l(t)}{dt}) + B_l \omega_l(t)}{\eta_g K_g} = \tau_m(t) \quad (1.1.16)$$

Collecting the coefficients in terms of the load shaft velocity and acceleration gives

$$(\eta_g K_g^2 J_m + J_l)\frac{d\omega_l(t)}{dt} + (\eta_g K_g^2 B_m + B_l)\omega_l(t) = \eta_g K_g \tau_m(t) \quad (1.1.17)$$

Defining the following terms:

$$J_{eq} = \eta_g K_{g2} J_m + J_l \quad (1.1.18)$$

$$B_{eq} = \eta_g K_{g2} B_m + B_l \quad (1.1.19)$$

simplifies the equation as:

$$J_{eq} \frac{d\omega_l(t)}{dt} + B_{eq}\omega_l(t) = \eta_g K_g \tau_m(t) \quad (1.1.20)$$

1.1.1.3 Combining the Electrical and Mechanical Equations

In this section the electrical equation derived in Section 1.1.1.1 and the mechanical equation found in Section 1.1.1.2 are brought together to get an expression that represents the load shaft speed in terms of the applied motor voltage. The motor torque is proportional to the voltage applied and is described as

$$\tau_m(t) = \eta_m k_t I_m(t) \quad (1.1.21)$$

where k_t is the current-torque constant ($N.m/A$), η_m is the motor efficiency, and I_m is the armature current. See [6] for more details on the SRV02 motor specifications.

We can express the motor torque with respect to the input voltage $V_m(t)$ and load shaft speed $\omega_l(t)$ by substituting the motor armature current given by equation 1.1.5 in Section 1.1.1.1, into the current-torque relationship given in equation 1.1.21:

$$\tau_m(t) = \frac{\eta_m k_t \left(V_m(t) - k_m \omega_m(t)\right)}{R_m} \quad (1.1.22)$$

To express this in terms of V_m and ω_l, insert the motor-load shaft speed equation 1.1.15, into 1.1.21 to get:

$$\tau_m(t) = \frac{\eta_m k_t \left(V_m(t) - k_m K_g \omega_l(t)\right)}{R_m} \quad (1.1.23)$$

If we substitute (1.1.23) into (1.1.20), we get:

$$J_{eq}\left(\frac{d}{dt} w_l(t)\right) + B_{eq} w_l(t) = \frac{\eta_g K_g \eta_m k_t \left(V_m(t) - k_m K_g \omega_l(t)\right)}{R_m} \quad (1.1.24)$$

After collecting the terms, the equation becomes

$$\left(\frac{d}{dt} w_l(t)\right) J_{eq} + \left(\frac{k_m \eta_g K_g^2 \eta_m k_t}{R_m} + B_{eq}\right)\omega_l(t) = \frac{\eta_g K_g \eta_m k_t V_m(t)}{R_m} \quad (1.1.25)$$

This equation can be re-written as:

$$\left(\frac{d}{dt}w_l(t)\right) J_{eq} + B_{eq,v}\omega_l(t) = A_m V_m(t) \tag{1.1.26}$$

where the equivalent damping term is given by:

$$B_{eq,v} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_{eq} R_m}{R_m} \tag{1.1.27}$$

and the actuator gain equals

$$A_m = \frac{\eta_g K_g \eta_m k_t}{R_m} \tag{1.1.28}$$

1.1.2 Modeling Using Experiments

In Section 1.1.1 you learned how the system model can be derived from the first-principles. A linear model of a system can also be determined purely experimentally. The main idea is to experimentally observe how a system reacts to different inputs and change structure and parameters of a model until a reasonable fit is obtained. The inputs can be chosen in many different ways and there are a large variety of methods. In Sections 1.1.2.1 and 1.1.2.2, two methods of modeling the SRV02 are outlined: (1) frequency response and, (2) bump test.

1.1.2.1 Frequency Response

In Figure 1.2, the response of a typical first-order time-invariant system to a sine wave input is shown. As it can be seen from the figure, the input signal (u) is a sine wave with a fixed amplitude and frequency. The resulting output (y) is also a sinusoid with the *same* frequency but with a different amplitude. By varying the frequency of the input sine wave and observing the resulting outputs, a Bode plot of the system can be obtained as shown in Figure 1.3.

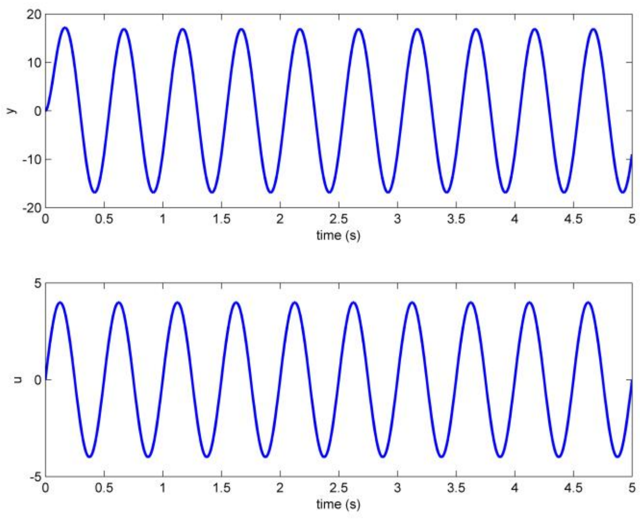

Figure 1.2: Typical frequency response

The Bode plot can then be used to find the steady-state gain, i.e. the DC gain, and the time constant of the system. The cuttoff frequency, ω_c, shown in Figure 1.3 is defined as the frequency where the gain is 3 dB less than the maximum gain (i.e. the DC gain). When working in the linear non-decibel range, the 3 dB frequency is defined as the frequency where the gain is $\frac{1}{\sqrt{2}}$, or about 0.707, of the maximum gain. The cutoff frequency is also known as the bandwidth of the system which represents how fast the system responds to a given input.

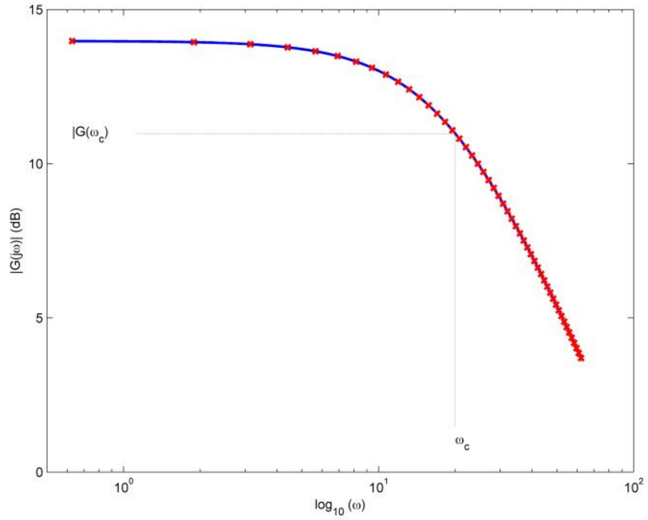

Figure 1.3: Magnitude Bode plot

The magnitude of the frequency response of the SRV02 plant transfer function given in equation 1.1.1 is defined as:

$$|G_{wl,v}(w)| = \left| \frac{\Omega_l(\omega\, j)}{V_m(\omega\, j)} \right| \qquad (1.1.29)$$

where ω is the frequency of the motor input voltage signal V_m. We know that the transfer function of the system has the generic first-order system form given in Equation 1.1.1. By substituting $s = j\, w$ in this equation, we can find the frequency response of the system as:

$$\frac{\Omega_l(\omega\, j)}{V_m(\omega\, j)} = \frac{K}{\tau \omega\, j + 1} \qquad (1.1.30)$$

Then, the magnitude of it equals

$$|G_{wl,v}(\omega)| = \frac{K}{\sqrt{1 + \tau^2\, \omega^2}} \qquad (1.1.31)$$

Let's call the frequency response model parameters $K_{e,f}$ and $\tau_{e,f}$ to differentiate them from the nominal model parameters, K and τ, used previously. The steady-state gain or the DC gain (i.e. gain at zero frequency) of the model is:

$$K_{e,f} = |G_{wl,v}(0)| \quad (1.1.32)$$

1.1.2.2 Bump Test

The bump test is a simple test based on the step response of a stable system. A step input is given to the system and its response is recorded. As an example, consider a system given by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \quad (1.1.33)$$

The step response shown in Figure 1.4 is generated using this transfer function with $K = 5$ rad/V.s and $\tau = 0.05$ s.

The step input begins at time t_0. The input signal has a minimum value of u_{min} and a maximum value of u_{max}. The resulting output signal is initially at y_0. Once the step is applied, the output tries to follow it and eventually settles at its steady-state value y_{ss}. From the output and input signals, the steady-state gain is

$$K = \frac{\Delta y}{\Delta u} \quad (1.1.34)$$

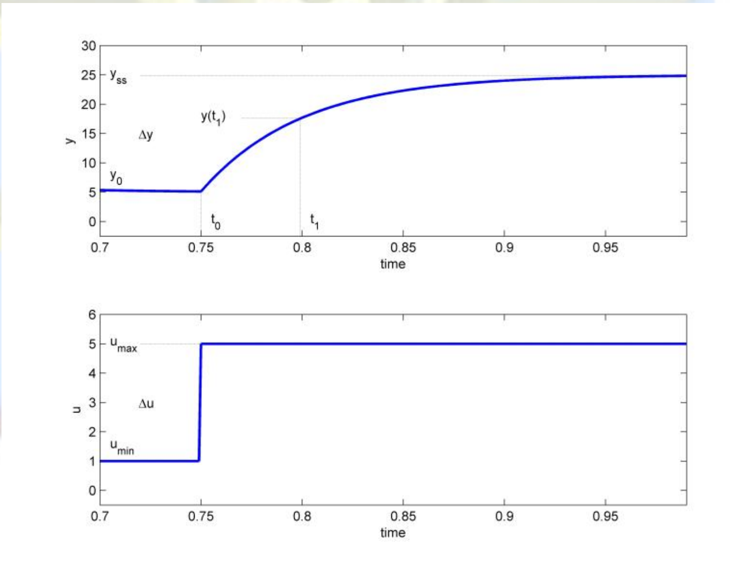

Figure 1.4: Input and output signal used in the bump test method

where $\Delta y = y_{ss} - y_0$ and $\Delta u = u_{max} - u_{min}$. In order to find the model time constant, τ, we can first calculate where the output is supposed to be at the time constant from:

$$y(t_1) = 0.632 y_{ss} + y_0 \quad (1.1.35)$$

Then, we can read the time t_1 that corresponds to $y(t_1)$ from the response data in Figure 1.4. From the figure we can see that the time t_1 is equal to:

$$t_1 = t_0 + \tau \quad (1.1.36)$$

From this, the model time constant can be found as:

$$\tau = t_1 - t_0 \quad (1.1.37)$$

Going back to the SRV02 system, a step input voltage with a time delay t_0 can be expressed as follows in the Laplace domain:

$$V_m(s) = \frac{A_v\, e^{(-s\, t_0)}}{s} \quad (1.1.38)$$

where A_v is the amplitude of the step and t_0 is the step time (i.e. the delay). If we substitute this input into the system transfer function given in Equation (1.1.1), we get:

$$\Omega_l(s) = \frac{K A_v\, e^{(-s\, t_0)}}{(\tau\, s + 1)\, s} \quad (1.1.39)$$

We can then find the SRV02 load speed step response, $w_l(t)$, by taking inverse Laplace of this equation. Here we need to be careful with the time delay t_0 and note that the initial condition is $\omega_l(0^-) = \omega_l(t_0)$.

$$\omega_l(t) = K\, A_v \left(1 - e^{\left(-\frac{t-t_0}{\tau}\right)}\right) + \omega_l\,(t_0) \quad (1.1.40)$$

1.2 Pre-Lab Questions

Before you start the lab experiments given in Section 1.3, you should study the background materials provided in Section 1.1 and work through the questions in this Section.

1. In Section 1.1.1.3 we obtained an equation (1.1.26) that described the dynamic behavior of the load shaft speed as a function of the motor input voltage. Starting from this equation, find the transfer function $\frac{\Omega_l(s)}{V_m(s)}$.

Answer 1.2.1

Outcome	Solution
A-1	Taking the Laplace transform of the equations and assuming $\omega_l(0^-) = 0$ gives
	$J_{eq} s \Omega_l(s) + B_{eq,v} \Omega_l(s) = A_m V_m(s)$ (Ans.1.2.1)
A-2	Solving for $\Omega_l(s)/V_m(s)$ gives the plant transfer function
	$\frac{\Omega_l(s)}{V_m(s)} = \frac{A_m}{J_{eq} s + B_{eq,v}}$ (Ans.1.2.2)

2. Express the steady-state gain (K) and the time constant (τ) of the process model (Equation (1.1.1)) in terms of the J_{eq}, $B_{eq,v}$, and A_m parameters.

Answer 1.2.2

Outcome Solution

A-1 We need to match the coefficients of the transfer function found in (Ans.1.2.2) to the coefficients of the transfer function in equation 1.1.1.

A-2 The time constant parameter is

$$\tau = \frac{J_{eq}}{B_{eq,v}} \quad \text{(Ans.1.2.3)}$$

and the steady-state gain is

$$K = \frac{A_m}{B_{eq,v}} \quad \text{(Ans.1.2.4)}$$

3. Calculate the $B_{eq,v}$ and A_m model parameters using the system specifications given in [6]. The parameters are to be calculated based on an SRV02-ET in the high-gear configuration.

Answer 1.2.3

Outcome Solution

A-2 The $B_{eq,v}$ viscous damping expression is given in Equation 1.1.27. All the parameters are defined in Reference [6] including the experimentally determined equivalent viscous damping parameter $B_{eq} = 0.015 N\ ms/rad$ (in the high-gear configuration). Substituting all the specifications into 1.1.27 gives

$$B_{eq,v} = 0.0844 \text{N m s / rad} \quad \text{(Ans.1.2.5)}$$

Evaluating the actuator gain expression in 1.1.28 with the SRV02 parameters gives

$$A_m = 0.129 \text{ N m/V} \quad \text{(Ans.1.2.6)}$$

4. Calculate the moment of inertia about the motor shaft. Note that $J_m = J_{tach} + J_{m,rotor}$ where J_{tach} and $J_{m,rotor}$ are the moment of inertia of the tachometer and the rotor of the SRV02 DC motor, respectively. Use the specifications given in [6].

Answer 1.2.4

Outcome Solution

A-2 The moment of inertia about the motor shaft equals

$$J_m = J_{tach} + J_{m,rotor} \quad \text{(Ans.1.2.7)}$$

Evaluating the above expression with the parameters outlined in [6] gives

$$J_m = 4.606251061 \times 10^{-7} \text{ kg m}^2 \quad \text{(Ans.1.2.8)}$$

5. The load attached to the motor shaft includes a 24-tooth gear, two 72-tooth gears, and a single 120-tooth gear along with any other external load that is attached to the load shaft. Thus, for the gear moment of inertia J_g and the external load moment of inertia $J_{l,ext}$, the load inertia is $J_l = J_g + J_{l,ext}$. Using the specifications given in [6] find the total moment of inertia J_g from the gears . **Hint:** Use the definition of moment of inertia for a disc $J_{disc} = \frac{mr^2}{2}$.

Answer 1.2.5

Outcome	Solution
A-2	The formula to calculate the moment of inertia of a disc is

$$J_{disc} = \frac{mr^2}{2} \qquad \text{(Ans.1.2.9)}$$

where m is the mass and r is the radius. Assuming the gears are discs and using the parameters given in Reference [6], the moment of inertia of the 24-tooth, 72-tooth, and 120-tooth gears are

$$J_{24} = 1.01 \times 10^{-7} \text{ kg m}^2 \qquad \text{(Ans.1.2.10)}$$

$$J_{72} = 5.44 \times 10^{-6} \text{ kg m}^2 \qquad \text{(Ans.1.2.11)}$$

and

$$J_{120} = 4.18 \times 10^{-5} \text{ kg m}^2 \qquad \text{(Ans.1.2.12)}$$

The total moment of inertia from the gears is

$$J_g = J_{24}(120/24)^2 + 2J_{72} + J_{120} \qquad \text{(Ans.1.2.13)}$$

which equals

$$J_g = 5.52 \times 10^{-5} \text{ kg m}^2 \qquad \text{(Ans.1.2.14)}$$

6. Assuming the disc load is attached to the load shaft, calculate the inertia of the disc load, $J_{ext,l}$, and the total load moment of inertia, J_l.

Answer 1.2.6

Outcome	Solution
A-2	Using the formula in Ans.1.2.9 with the m_b and r_b disc load parameters found in Reference [6], the external load moment of inertia equals

$$J_{l,ext} = 5.00 \times 10^{-5} \text{ kg m}^2 \qquad \text{(Ans.1.2.15)}$$

Using $J_l = J_g + J_{l,ext}$, the total load moment of inertia is

$$J_l = 1.05 \times 10^{-4} \text{ kg m}^2 \qquad \text{(Ans.1.2.16)}$$

7.Evaluate the equivalent moment of inertia J_{eq}.

Answer 1.2.7

Outcome Solution

A-2 Using Equation 1.1.18 with the gear train and motor specifications listed in Reference [6] and the load inertia found in 1.2.6, the equivalent moment of inertia acting on the SRV02 motor shaft is

$$J_{eq} = 0.00214 \text{ kg m}^2. \quad \text{(Ans.1.2.17)}$$

8. Calculate the steady-state model gain K and time constant τ. These are the *nominal model parameters* and will be used to compare with parameters that are later found experimentally.

Answer 1.2.8

Outcome Solution

A-2 Using equations Ans.1.2.3 and Ans.1.2.4 with the $B_{eq,v}$, A_m, and J_{eq} parameters found in equations Ans.1.2.5, Ans.1.2.6, and Ans.1.2.17, the steady-state gain is

$$K = 1.53 \text{ rad/(V s)} \quad \text{(Ans.1.2.18)}$$

and the model time constant is

$$\tau = 0.0253 \text{ s} \quad \text{(Ans.1.2.19)}$$

9. Referring to Section 1.1.2.1, find the expression representing the time constant τ of the frequency response model given in Equation 1.1.31. Begin by evaluating the magnitude of the transfer function at the cutoff frequency ω_c.

Answer 1.2.9

Outcome Solution

A-1 By definition, the DC gain drops 3 dB (or $\frac{1}{\sqrt{2}}$) at this frequency. Therefore,

$$|G_{wl,v}(w_c)| = \frac{1}{2}\,|G_{wk,v}(0)|\,\sqrt{2} \quad \text{(Ans.1.2.20)}$$

A-2 Applying this to the SRV02 frequency response magnitude in 1.1.31

above gives:

$$\frac{1}{2}\,|G_{wl,v}(0)|\,\sqrt{2} = \frac{G_{wl,v}(0)}{\sqrt{1+\tau_{e,f}^2\,\omega_c^2}} \quad \text{(Ans.1.2.21)}$$

We can then solve for the time constant as:

$$\tau_{e,f} = \frac{1}{|\omega_c|} \quad \text{(Ans.1.2.22)}$$

10. Referring to Section 1.1.2.2, find the steady-state gain of the step response and compare it with Equation 1.1.34. **Hint:** The the steady-state value of the load shaft speed can be defined as $\omega_{l,ss} = \lim_{t\to\infty} \omega_l(t)$.

Answer 1.2.10

Outcome	Solution
A-2	Using the definition of the steady-state value of the load shaft

$$\omega_{l,ss} = \lim_{t\to\infty} \omega_l(t) \quad \text{(Ans.1.2.23)}$$

The limit of the servo step response given in (1.1.40) is

$$\omega_{l,ss} = K A_v + \omega_l(t_0) \quad \text{(Ans.1.2.24)}$$

and the steady-state gain is

$$K = \frac{\omega_{l,ss} - \omega_l(t_0)}{A_v} \quad \text{(Ans.1.2.25)}$$

A-3 This is consistent with the $\Delta y/\Delta u$ relationship in Equation 1.1.34.

11.Evaluate the step response given in equation 1.1.40 at $t = t_0 + \tau$ and compare it with Equation 1.1.34.

Answer 1.2.11

Outcome	Solution
A-2	Substituting $t = t_0 + \tau$ in equation 1.1.40 gives the load shaft rate

$$\omega_l(t_0 + \tau) = K A_v (1 - e^{-1}) + \omega_l(t_0) \quad \text{(Ans.1.2.26)}$$

A-3 This is consistent with the $y(t_1)$ expression in equation 1.1.34.

1.4 System Requirements

Before you begin this laboratory make sure:

- QUARC®is installed on your PC, as described in Reference [1].
- You have a QUARC compatible data-aquisition (DAQ) card installed in your PC. For a listing of compliant DAQ cards, see Reference [5].
- SRV02 and amplifier are connected to your DAQ board as described Reference [6].

SRV02 POSITION CONTROL

The objective of this laboratory is to develop feedback systems that control the position of the rotary servo load shaft. Using the proportional-integral-derivative (PID) family, controllers are designed to meet a set of specifications.

Topics Covered

- Design of a proportional-velocity (PV) controller for position control of the servo load shaft to meet certain time-domain requirements.
- Actuator saturation.
- Design of a proportional-velocity-integral (PIV) controller to track a ramp reference signal.
- Simulation of the PV and PIV controllers using the developed model of the plant to ensure the specifications are met without any actuator saturation.
- Implementation of the controllers on the Quanser SRV02 device to evaluate their performance.

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- Data acquisition device (e.g. Q2-USB), the power amplifier (e.g. VoltPAQ-X1), and the main components of the SRV02 (e.g. actuator, sensors), as described in References [2], [4], and [6], respectively.
- Wiring and operating procedure of the SRV02 plant with the amplifier and data-aquisition (DAQ) device, as discussed in Reference [6].
- Transfer function fundamentals, e.g. obtaining a transfer function from a differential equation.
- Laboratory described in Appendix A to get familiar with using QUARCᴿwith the SRV02.

2.1 Background

2.1.1 Desired Position Control Response

The block diagram shown in Figure 2.1 is a general unity feedback system with compensator (controller) C(s) and a transfer function representing the plant, P(s). The measured output, Y(s), is supposed to track the reference signal R(s) and the tracking has to match to certain desired specifications.

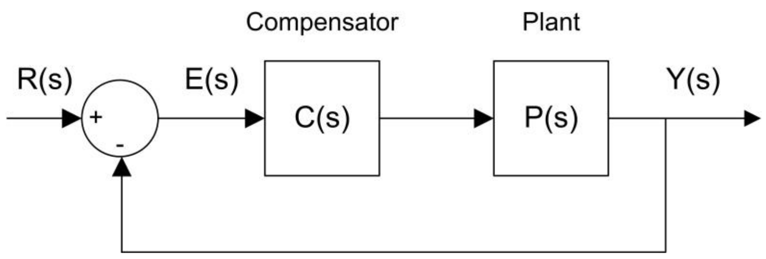

Figure 2.1: Unity feedback system.

The output of this system can be written as:

$$Y(s) = C(s)P(s)\,(R(s) - Y(s)) \qquad (2.1.1)$$

By solving for $Y(s)$, we can find the closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)} \qquad (2.1.2)$$

Recall in Laboratory: SRV02 Modelling Section 1, the SRV02 voltage-to-speed transfer function was derived. To find the voltage-to-position transfer function, we can put an integrator ($1/s$) in series with the speed transfer function (effectively integrating the speed output to get position). Then, the resulting open-loop voltage-to-load gear position transfer function becomes:

$$P(s) = \frac{K}{s\,(\tau s + 1)} \qquad (2.1.3)$$

As you can see from this equation, the plant is a second order system. In fact, when a second order system is placed in series with a proportional compensator in the feedback loop as in Figure 2.1, the resulting closed-loop transfer function can be expressed as:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\,\omega_n\,s + \omega_n^2} \qquad (2.1.4)$$

where ω_n is the natural frequency and ζ is the damping ratio. This is called the *standard second-order* transfer function. Its response proporeties depend on the values of ω_n and ζ.

2.1.1.1 Peak Time and Overshoot

Consider a second-order system as shown in Equation 2.1.4 subjected to a step input given by

$$R(s) = \frac{R_0}{s} \qquad (2.1.5)$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 2.2, where the red trace is the response (output), $y(t)$, and the blue trace is the step input $r(t)$. The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 2.2, the percent overshoot is found using

$$PO = \frac{100\,(y_{max} - R_0)}{R_0} \qquad (2.1.6)$$

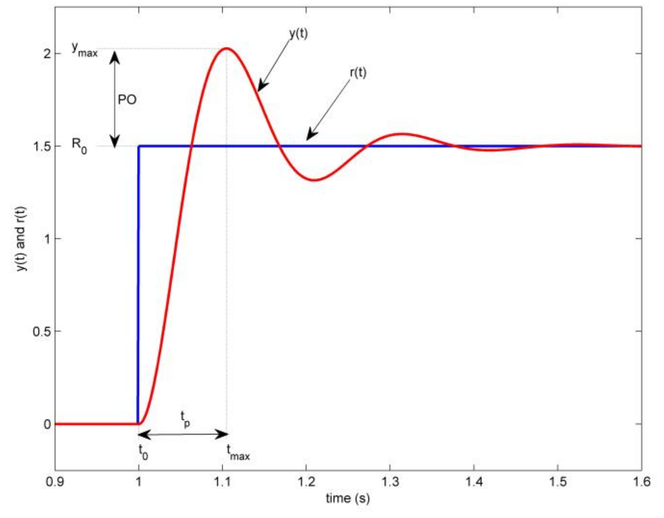

Figure 2.2: Standard second-order step response.

From the initial step time, t_0, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0 \qquad (2.1.7)$$

This is called the *peak time* of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100\, e^{\left(-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}\right)} \qquad (2.1.8)$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as:

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \qquad (2.1.9)$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

2.1.1.2 Steady State Error

Steady-state error is illustrated in the ramp response given in Figure 2.3 and is denoted by the variable e_{ss}. It is the difference between the reference input and output signals after the system response has settled. Thus, for a time t when the system is in steady-state, the steady-state error equals

$$e_{ss} = r_{ss}(t) - y_{ss}(t) \qquad (2.1.10)$$

where $r_{ss}(t)$ is the value of the steady-state input and $y_{ss}(t)$ is the steady-state value of the output.

We can find the error transfer function $E(s)$ in Figure 2.1 in terms of the reference $R(s)$, the plant $P(s)$, and the compensator $C(s)$. The Laplace transform of the error is

$$E(s) = R(s) - Y(s) \qquad (2.1.11)$$

Solving for $Y(s)$ from equation 2.1.3 and substituting it in equation 2.1.11 yields

$$E(s) = \frac{R(s)}{1 + C(s)\,P(s)} \qquad (2.1.12)$$

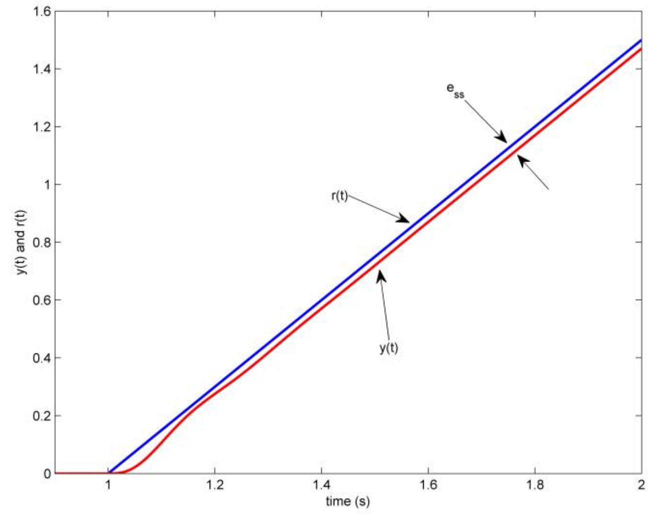

Figure 2.3: Steady-state error in ramp response.

We can find the the steady-state error of this system using the final-value theorem:

$$e_{ss} = \lim_{s \to 0} sE(s) \qquad (2.1.13)$$

In this equation, we need to substitute the transfer function for E(s) from 2.1.12. The E(s) transfer function requires, R(s), C(s) and P(s). For simplicity, let C(s)=1 as a compensator. The P(s) and R(s) were given by equations 2.1.3 and 2.1.5, respectively. Then, the error becomes:

$$E(s) = \frac{R_0}{s\left(1 + \frac{K}{s\,(\tau s+1)}\right)} \qquad (2.1.14)$$

Applying the final-value theorem gives

$$e_{ss} = R_0 \left(\lim_{s \to 0} \frac{(\tau\, s + 1)\; s}{\tau\, s^2 + s + K} \right) \qquad (2.1.15)$$

When evaluated, the resulting steady-state error due to a step response is

$$e_{ss} = 0 \qquad (2.1.16)$$

Based on this zero steady-state error for a step input, we can conclude that the SRV02 is a *Type 1* system.

2.1.1.3 SRV02 Position Control Specifications

The desired time-domain specifications for controlling the position of the SRV02 load shaft are:

$$e_{ss} = 0 \quad (2.1.17) \quad t_p = 0.20 \text{ s} \quad (2.1.18)$$

and

$$PO = 5.0 \, \% \quad (2.1.19)$$

Thus, when tracking the load shaft reference, the transient response should have a peak time less than or equal to 0.20 seconds, an overshoot less than or equal to 5 %, and the steady-state response should have no error.

2.1.2 PV Controller Design

2.1.2.1 Closed Loop Transfer Function

The proportional-velocity (PV) compensator to control the position of the SRV02 has the following structure

$$V_m(t) = k_p \, (\theta_d(t) - \theta_l(t)) - k_v \left(\frac{d}{dt} \, \theta_l(t) \right) \quad (2.1.20)$$

where k_p is the proportional control gain, k_v is the velocity control gain, $\theta_d(t)$ is the setpoint or reference load shaft angle, $\theta_l(t)$ is the measured load shaft angle, and $V_m(t)$ is the SRV02 motor input voltage. The block diagram of the PV control is given in Figure 2.4. We need to find the closed-loop transfer function $\Theta_l(s)/\Theta_d(s)$ for the closed-loop

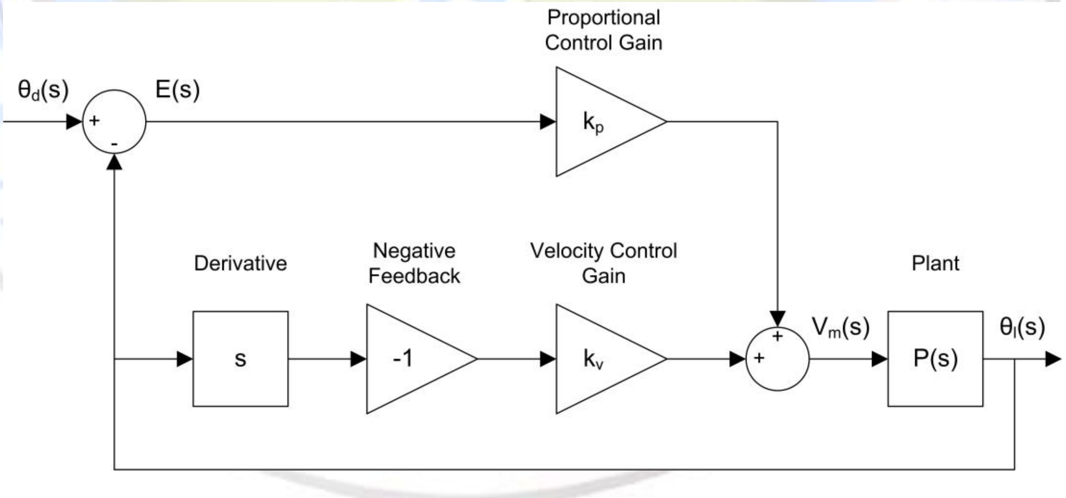

Figure 2.4: Block diagram of SRV02 PV position control.

position control of the SRV02. Taking the Laplace transform of equation 2.1.20 gives

$$V_m(s) = k_p \, (\Theta_d(s) - \Theta_l(s)) - k_v \, s\Theta_l(s) \quad (2.1.21)$$

From the Plant block in Figure 2.4 and equation 2.1.3, we can write

$$\frac{\Theta_l(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)} \tag{2.1.22}$$

Substituting equation 2.1.21 into 2.1.22 and solving for $\Theta_l(s)/\Theta_d(s)$ gives the SRV02 position closed-loop transfer function as:

$$\frac{\Theta_l(s)}{\Theta_d(s)} = \frac{K k_p}{\tau s^2 + (1 + K k_v) s + K k_p} \tag{2.1.23}$$

2.1.2.2 Controller Gain Limits

In control design, a factor to be considered is saturation. This is a nonlinear element and is represented by a saturation block as shown in Figure 2.5. In a system like the SRV02, the computer calculates a numeric control voltage value. This value is then converted into a voltage, $V_{dac}(t)$, by the digital-to-analog converter of the dataacquisition device in the computer. The voltage is then amplified by a power amplifier by a factor of K_a. If the amplified voltage, $V_{amp}(t)$, is greater than the maximum output voltage of the amplifier or the input voltage limits of the motor (whichever is smaller), then it is saturated (limited) at V_{max}. Therefore, the input voltage $V_m(t)$ is the effective voltage being applied to the SRV02 motor. The limitations of the actuator must be taken into account when designing a controller. For instance, the voltage entering the SRV02 motor should never exceed

$$V_{max} = 10.0 \text{ V} \tag{2.1.24}$$

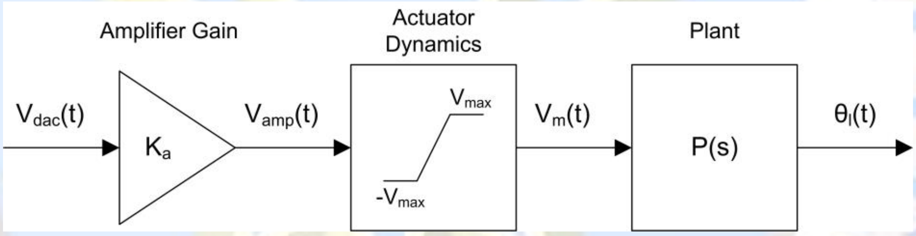

Figure 2.5: Actuator saturation.

2.1.2.3 Ramp Steady State Error Using PV Control

From our previous steady-state analysis, we found that the closed-loop SRV02 system is a Type 1 system. In this section, we will investigate the steady-state error due to a *ramp* input when using PV controller.

Given the following ramp setpoint (input)

$$R(s) = \frac{R_0}{s^2} \tag{2.1.25}$$

we can find the error transfer function by substituting the SRV02 closed-loop transfer function in equation 2.1.23 into the formula given in 2.1.11. Using the variables of the SRV02, this formula can be rewritten as $E(s) = \Theta_d(s) - \Theta_l(s)$. After rearranging the terms we find:

$$E(s) = \frac{\Theta_d(s)\, s\,(\tau\, s + 1 + K\, k_v)}{\tau\, s^2 + s + K\, k_p + K\, k_v\, s} \quad (2.1.26)$$

Substituting the input ramp transfer function 2.1.25 into the $\Theta_d(s)$ variable gives

$$E(s) = \frac{R_0\ (\tau\, s + 1 + K\, k_v)}{s\ (\tau\, s^2 + s + K\, k_p + K\, k_v\, s)} \quad (2.1.27)$$

2.1.3 PIV Controller

Adding an integral control can help eliminate any steady-state error. We will add an integral signal (middle branch in Figure 2.6) to have a proportional-integral-velocity (PIV) algorithm to control the position of the SRV02. The motor voltage will be generated by the PIV according to:

$$V_m(t) = k_p\ (\theta_d(d) - \theta_l(t)) + k_i \int (\theta_d(t) - \theta_l(t))\, dt - k_v\ \left(\frac{d}{dt}\, \theta_l(t)\right) \quad (2.1.28)$$

where k_i is the integral gain. We need to find the closed-loop transfer function $\Theta_l(s)/\Theta_d(s)$ for the closed-loop position control of the SRV02. Taking the Laplace transform of equation 2.1.28 gives

$$V_m(s) = \left(k_p + \frac{k_i}{s}\right)\ (\Theta_d(s) - \Theta_l(s)) - k_v\, s\, \Theta_l(s) \quad (2.1.29)$$

From the Plant block in Figure 2.6 and equation 2.1.3, we can write

$$\frac{\Theta_l(s)}{V_m(s)} = \frac{K}{(\tau\, s + 1)\ s} \quad (2.1.30)$$

Substituting equation 2.1.29 into 2.1.30 and solving for $\Theta_l(s)/\Theta_d(s)$ gives the SRV02 position closed-loop transfer function $\frac{\Theta_l(s)}{\Theta_d(s)} = \frac{K\ (k_p\, s + k_i)}{s^3\, \tau + (1 + K\, k_v)\, s^2 + K\, k_p\, s + K\, k_i}$ as: (2.1.31)

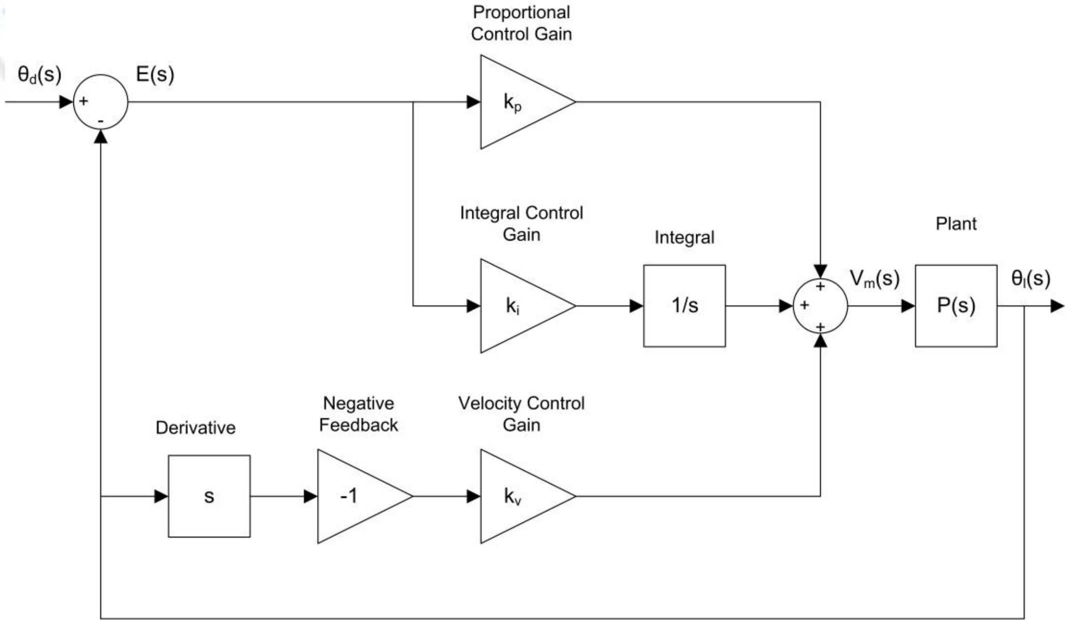

Figure 2.6: Block diagram of PIV SRV02 position control.

2.1.3.1 Ramp Steady-State Error using PIV Controller

To find the steady-state error of the SRV02 for a ramp input under the control of the PIV substitute the closed-loop transfer function from equation 2.1.31 into equation 2.1.11

$$E(s) = \frac{\Theta_d(s)\, s^2\ (\tau\, s + 1 + K\, k_v)}{s^3\, \tau + s^2 + K\, k_p\, s + K\, k_i + K\, k_v\, s^2} \quad (2.1.32)$$

Then, substituting the reference ramp transfer function 2.1.25 into the $\Theta_d(s)$ variable gives

$$E(s) = \frac{R_0\ (\tau\, s + 1 + K\, k_v)}{s^3\, \tau + s^2 + K\, k_p\, s + K\, k_i + K\, k_v\, s^2} \quad (2.1.33)$$

2.1.3.2 Integral Gain Design

It takes a certain amount of time for the output response to track the ramp reference with zero steady-state error. This is called the *settling time* and it is determined by the value used for the integral gain.

In steady-state, the ramp response error is constant. Therefore, to design an integral gain the velocity compensation (the V signal) can be neglected. Thus, we have a PI controller left as:

$$V_m(t) = k_p\,(\theta_d(t) - \theta_l(t)) + k_i \int (\theta_d(t) - \theta_l(t))\, dt \quad (2.1.34)$$

When in steady-state, the expression can be simplified to

$$V_m(t) = k_p\, e_{ss} + k_i \int_0^{t_i} e_{ss}\, dt \quad (2.1.35)$$

where the variable t_i is the integration time.

2.2 Pre-Lab Questions

Before you start the lab experiments given in Section 2.3, you should study the background materials provided in Section 2.1 and work through the questions in this Section.

1. Calculate the maximum overshoot of the response (in radians) given a step setpoint of 45 degrees and the overshoot specification given in Section 2.1.1.3.
   Hint: By substituting $y_{max} = \theta(t_p)$ and step setpoint $R_0 = \theta_d(t)$ into equation 2.1.6, we can obtain $\theta(t_p) = \theta_d(t)\ \left(1 + \frac{PO}{100}\right)$. Recall that the desired response specifications include 5% overshoot.

Answer 2.2.1

Outcome Solution

A-2 Substituting a step reference of $\theta_d(t) = 0.785$ rad and $PO = 5\ \%$ into this equation gives the maximum overshoot as $\theta(t_p) = 0.823$ rad.

2. The SRV02 closed-loop transfer function was derived in equation 2.1.23 in Section 2.1.2.1. Find the control gains k_p and k_v in terms of ω_n and ζ. **Hint:** Remember the standard second order system equation.

Answer 2.2.2

Outcome Solution

A-1 The characteristic equation of the SRV02 closed-loop transfer function in 2.1.7 is

$$\tau s^2 + (1 + K k_v)s + K k_p \quad \text{(Ans.2.2.1)}$$

and can be re-structured into the form

$$s^2 + \frac{(1 + K k_v)\ s}{\tau} + \frac{K k_p}{\tau} \quad \text{(Ans.2.2.2)}$$

Equating this with the standard second order system equation gives the expressions

$$\frac{K k_p}{\tau} = \omega_n^2 \quad \text{(Ans.2.2.3)}$$

and

$$\frac{1 + K k_v}{\tau} = 2\,\zeta\,\omega_n \quad \text{(Ans.2.2.4)}$$

A-2 Solve for k_p and k_v to obtain the control gains equations

$$k_p = \frac{\omega_n^2\,\tau}{K} \quad \text{(Ans.2.2.5)}$$

and the velocity gain is

$$k_v = \frac{2\zeta\omega_n\tau - 1}{K} \quad \text{(Ans.2.2.6)}$$

3. Calculate the minimum damping ratio and natural frequency required to meet the specifications given in Section 2.1.1.3.

Answer 2.2.3

Outcome

A-2 Substitute the percent overshoot specifications given in 2.1.19 into Equation 2.1.8 to get the required damping ratio

$$\zeta = 0.690 \quad \text{(Ans.2.2.7)}$$

Using this result and the desired peak time, given in 2.1.18, with Equation 2.1.9 gives the minimum natural frequency needed

$$\omega_n = 21.7 \text{ rad/s} \quad \text{(Ans.2.2.8)}$$

4. Based on the nominal SRV02 model parameters, K and τ, found in Laboratory 1: SRV02 Modeling, calculate the control gains needed to satisfy the time-domain response requirements given in Section 2.1.1.3.

Answer 2.2.4

Outcome **Solution**

A-2 Using the model parameters

$$K = 1.53 \text{ rad/(V s)} \quad \text{(Ans.2.2.9)}$$

and

$$\tau = 0.0254 \text{ s} \quad \text{(Ans.2.2.10)}$$

as well as the desired natural frequency found in Ans.2.2.8 with Equation Ans.2.2.5, generates the proportional control gain

$$k_p = 7.82 \text{ V/rad} \quad \text{(Ans.2.2.11)}$$

Similarly, the velocity control gain is obtained by substituting the model parameters given above with the minimum damping ratio specification, in Ans.2.2.7, into Equation Ans.2.2.6

$$k_v = -0.157 \text{ V s/rad} \quad \text{(Ans.2.2.12)}$$

Thus, when these gains are used with the PV controller, the position response of the load gear on an SRV02 with a disc load will satisfy the specifications listed in 2.1.1.3.

5. In the PV controlled system, for a reference step of $\pi/4$ (i.e. 45 degree step) starting from $\Theta_l(t) = 0$ position, calculate the *maximum* proportional gain that would lead to providing the maximum voltage to the motor. Ignore the velocity control ($k_v = 0$). Can the desired specifications be obtained based on this maximum available gain and what you calculated in question 4?

Answer 2.2.5

Outcome

A-1 The maximum proportional gain leads to providing the maximum voltage to the motor. Therefore, the PV control in 3.1.15 becomes

$$10.0 = \frac{1}{4} k_p \pi \quad \text{(Ans.2.2.13)}$$

A-2 after substituting the maximum SRV02 input voltage 2.1.24 for $V_m(t)$, the reference step of $\pi/4$, and $k_v = 0$ (to ignore the velocity control). Thus, the maximum proportional gain before saturating the SRV02 motor is

$$k_{p,max} = 12.7 \left[\frac{V}{rad}\right] \quad \text{(Ans.2.2.14)}$$

A-3 The proportional gain designed in Ans.2.2.11 is below $k_{p,max}$, therefore the desired specifications, can still be obtained.

6. For the PV controlled closed-loop system, find the steady-state error and evaluate it numerically given a ramp with a slope of $R_0 = 3.36$ rad/s. Use the control gains found in question 4.

Answer 2.2.6

Outcome **Solution**

A-1 Applying the final-value theorem to the error transfer function yields the expression

$$e_{ss} = \lim_{s \to 0} \frac{R_0\,(\tau\,s + 1 + K\,k_v)}{\tau\,s^2 + s + K\,k_p + K\,k_v\,s} \quad \text{(Ans.2.2.15)}$$

A-2 When evaluated, the resulting steady-state error is

$$e_{ss} = \frac{R_0\,(1 + K\,k_v)}{K\,k_p} \quad \text{(Ans.2.2.16)}$$

The steady-state error is a constant, which is as expected since the closed-loop SRV02 position system is Type 1. Evaluating the expression with the reference slope of 3.36 rad/s, the model gain parameter $K = 1.53$, the proportional and velocity gains $k_p = 7.82$ and $k_v = 0.157$, gives the steady-state error

$$e_{ss} = 0.214\ [rad] \quad \text{(Ans.2.2.17)}$$

7. What should be the integral gain k_i so that when the SRV02 is supplied with the maximum voltage of $V_{max} = 10V$ it can eliminate the steady-state error calculated in question 6 in 1 second? **Hint:** Start from equation 2.1.35 and use $t_i = 1$, $V_m(t) = 10$, the k_p you found in question 4 and e_{ss} found in question 6. Remember that e_{ss} is constant.

Answer 2.2.7

Outcome

A-2 Since e_{ss} is constant, evaluating the integral in Equation 2.1.35 yields

$$V_m(t) = k_p\,e_{ss} + k_i\,t_i\,e_{ss} \quad \text{(Ans.2.2.18)}$$

Then, the integral gain is

$$k_i = \frac{V_m(t) - k_p\, e_{ss}}{t_i\, e_{ss}} \qquad \text{(Ans.2.2.19)}$$

By substituting $t_i = 1.0 sec$, the maximum SRV02 voltage $V_m(t) = 10V$, $k_p = 7.82$ and the PV control steady-state error $e_{ss} = 0.214$ we find

$$k_i = 38.9 \text{ V/(rad s)} \qquad \text{(Ans.2.2.20)}$$

2.3 Lab Experiments

The main goal of this laboratory is to explore position control of the SRV02 load shaft using PV and PIV controllers.

In this laboratory, you will conduct three experiments:

1. Step response with PV controller,
2. Ramp response with PV controller, and
3. Ramp response with no steady-state error.

You will need to design the third experiment yourself. In each experiment, you will first simulate the closed-loop response of the system. Then, you will implement the controller using the SRV02 hardware and software to compare the real response to the simulated one.

2.3.1 Step Response Using PV Controller

2.3.1.1 Simulation

First, you will simulate the closed-loop response of the SRV02 with a PV controller to step input. Our goals are to confirm that the desired reponse specifications in an ideal situation are satisfied and to verify that the motor is not saturated. Then, you will explore the effect of using a high-pass filter, instead of a direct derivative, to create the velocity signal V in the controller.

Experimental Setup

The *s_srv02_pos* Simulink® diagram shown in Figure 2.7 will be used to simulate the closed-loop position control response with the PV and PIV controllers. The SRV02 Model uses a *Transfer Fcn* block from the Simulink® library. The PIV Control subsystem contains the PIV controller detailed in Section 2.1.3. When the integral gain is set to zero, it essentially becomes a PV controller.

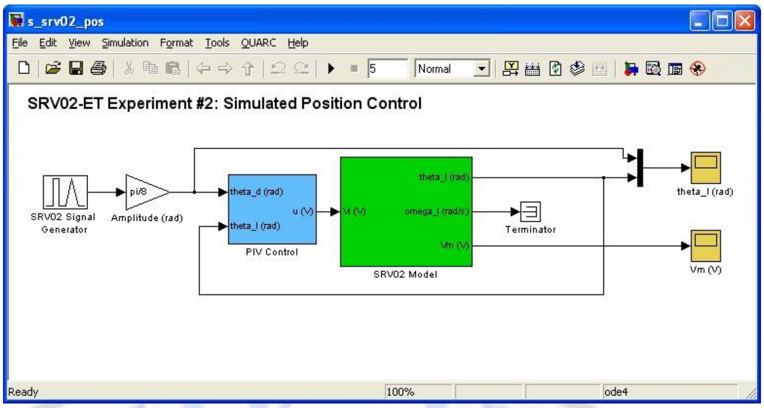

Figure 2.7: Simulink model used to simulate the SRV02 closed-loop position response.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your SRV02 setup. If they have not been configured already, then you need to go to Section 2.4.2 to configure the lab files first.

Closed-loop Response with the PV Controller

1. Enter the proportional and velocity control gains found in Pre-Lab question 4 in Matlab^{r}as *kp* and *kv*.
2. To generate a step reference, ensure the *SRV02 Signal Generator* is set to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.4 Hz
3. In the Simulink^{r}diagram, set the *Amplitude (rad)* gain block to $\pi/8(rad)$ to generate a step with an amplitude of 45 degrees (i.e., square wave goes between $\pm\pi/8$ which results in a step amplitude of $\pi/4$) .
4. Inside the *PIV Control* subsystem, set the *Manual Switch* to the upward position so the *Derivative block* is used.
5. Open the load shaft position scope, *theta_l (rad)*, and the motor input voltage scope, *Vm (V)*.
6. Start the simulation. By default, the simulation runs for 5 seconds. The scopes should be displaying responses similar to figures 2.8 and 2.9 Note that in the *theta_l* (rad) scope, the yellow trace is the setpoint position while the purple trace is the simulated position (generated by the *SRV02 Model* block). This simulation is called the *Ideal PV* response as it uses the PV compensator with the derivative block.

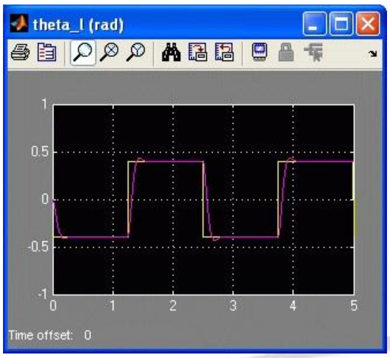

Figure 2.8: Ideal PV position response.

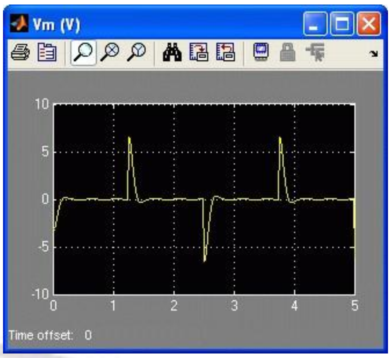

Figure 2.9: Ideal PV motor input voltage.

7. Generate a Matlab®figure showing the *Ideal PV* position response and the ideal input voltage. After each simulation run, each scope automatically saves their response to a variable in the Matlab®workspace. That is, the *theta_l (rad)* scope saves its response to the variable called data_pos and the *Vm (V)* scope saves its data to the *data_vm* variable. The *data_pos* variable has the following structure: *data_pos(:,1)* is the time vector, *data_pos(:,2)* is the setpoint, and *data_pos(:,3)* is the simulated angle. For the *data_vm* variable, *data_vm(:,1)* is the time and *data_vm(:,2)* is the simulated input voltage.

Answer 2.3.1

Outcome	Solution
K-3	The closed-loop position response with the straight derivative, i.e. the ideal response, is shown in Figure 2.10. This is generated using the *sample_meas_tp_os.m* script. To use this script, do the following: (a) Execute the *setup_srv02_exp02_pos.m* script with CONTROL_TYPE = 'AUTO_PV'. (b) Run the *s_srv02_pos* Simulink®model. (c) Run the *sample_meas_tp_os.m* script.

8.Measure the steady-state error, the percent overshoot and the peak time of the simulated response.

Does the response satisfy the specifications given in Section 2.1.1.3? **Hint:** Use the Matlab®*ginput* command to take measurements off the figure.

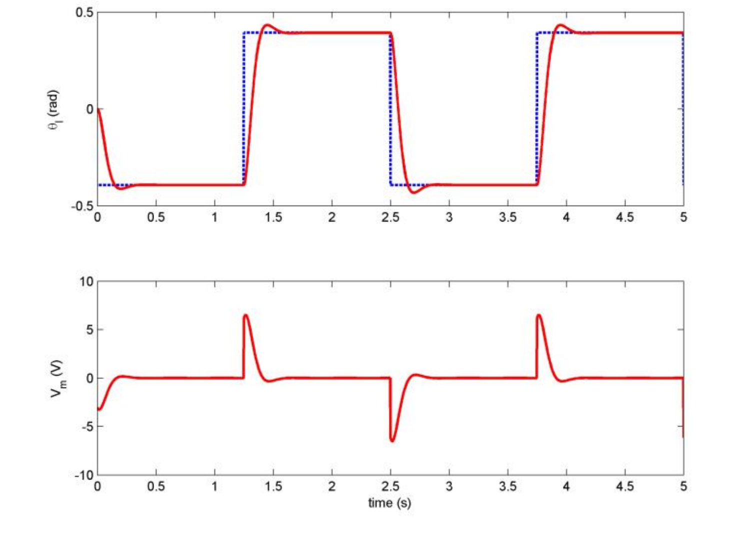

Figure 2.10: Ideal closed-loop PV position response.

Answer 2.3.2

Outcome Solution

K-1 Directly from the response shown in Figure 2.10, it is clear that the steady-state error is zero, thus

$$e_{ss} = 0 \quad \text{(Ans.2.3.1)}$$

Using Equation 2.1.7, the peak time of the response in Figure 2.10 is

$$t_p = 0.20 \text{ s} \quad \text{(Ans.2.3.2)}$$

Similarly, the percent overshoot is calculated using Equation 2.1.6 as

$$PO = 5.0\ \% \quad \text{(Ans.2.3.3)}$$

B-9 The response with the PV controller matches the specifications in Section 2.1.1.3 while maintaining a motor input voltage less than 10 V, i.e. the motor is not saturated. To find the peak time and percent overshoot of a response saved in *data_pos* automatically, run the *sample_meas_tp_os.m* script after running *s_srv02_pos*.

Using a High-pass Filter Instead of Direct Derivative

9. When implementing a controller on actual hardware, it is generally not advised to take the direct derivative of a measured signal. Any noise or spikes in the signal becomes amplified and gets multiplied by a gain and fed into the motor which may lead to damage. To remove any high-frequency noise components in the velocity signal, a low-pass filter is placed in

series with the derivative, i.e. taking the high-pass filter of the measured signal. However, as with a controller, the filter must also be tuned properly. In addition, the filter has some adverse affects. Go in the *PIV Control* block and set the Manual Switch block to the down position to enable the high-pass filter.

10. Start the simulation. The response in the scopes should still be similar to figures 2.8 and 2.9. This simulation is called the *Filtered PV* response as it uses the PV controller with the high-pass filter block.

11. Generate a Matlab[r] figure showing the *Filtered PV* position and input voltage responses.

Answer 2.3.3

Outcome	Solution
K-3	The Filtered PV step response is illustrated in Figure 2.11. This is generated by executing the *sample_meas_tp_os.m* script after running the *s_srv02_pos* simulation.

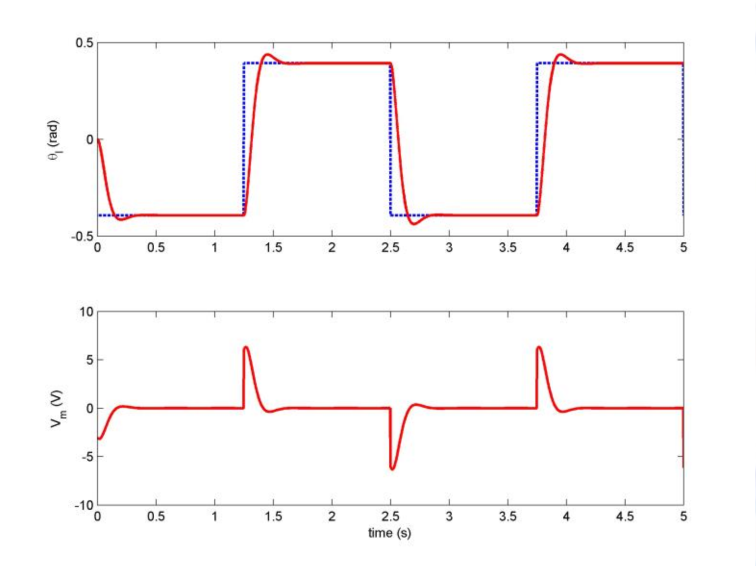

Figure 2.11: Filtered closed-loop PV position response.

12. Measure the steady-state error, peak time, and percent overshoot. Are the specifications still satisfied without saturating the actuator? Recall that the peak time and percent overshoot should not exceed the values given in Section 2.1.1.3. Discuss the changes from the ideal response. **Hint:** The different in the response is minor. Make sure you use *ginput* to take precise measurements.

Answer 2.3.4

Outcome Solution

K-1 As with the ideal response, the steady-state error of the filtered response in Figure 2.11 is

$$e_{ss} = 0 \qquad \text{(Ans.2.3.4)}$$

and the peak time is

$$t_p = 0.20 \text{ s} \qquad \text{(Ans.2.3.5)}$$

Thus, the PV Filtered response satisfies the error and peak time specifications given in Section 2.1.1.3. The percent overshoot of the response shown in Figure 2.11 is

$$PO = 5.76\ \% \qquad \text{(Ans.2.3.6)}$$

B-9 This exceeds the 5 % overshoot requirement and, as a result, not all the specifications are satisfied. To find the peak time and percentage overshoot of a response saved in *data_pos* automatically, run the *sample_meas_tp_os.m* script after running *s_srv02_pos*.

2.3.1.2 Implementing Step Response using PV Controller

In this experiment, we will control the angular position of the SRV02 load shaft, i.e. the disc load, using the PV controller. Measurements will then be taken to ensure that the specifications are satisfied.

Experimental Setup

The *q_srv02_pos* Simulink® diagram shown in Figure 2.12 is used to implement the position control experiments. The *SRV02-ET* subsystem contains QUARC blocks that interface with the DC motor and sensors of the SRV02 system, as discussed in Section A. The *PIV Control* subsystem implements the PIV controller detailed in Section 2.1.3, except a high-pass filter is used to obtain the velocity signal (as opposed to taking the direct derivative).

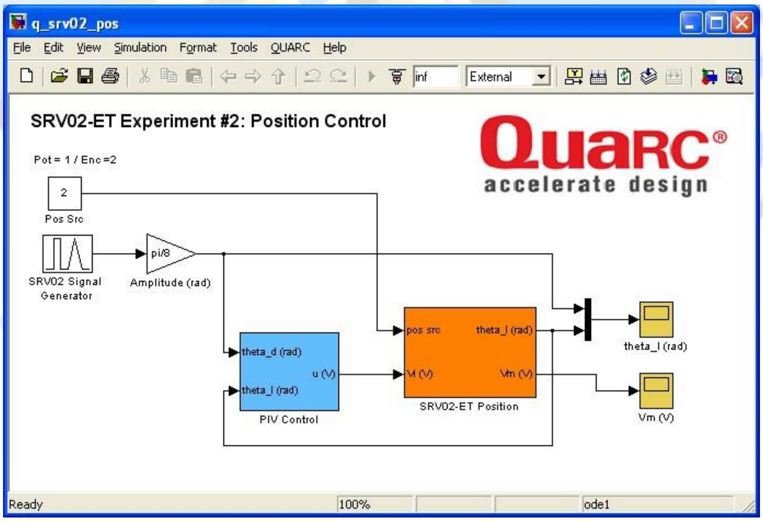

Figure 2.12: Simulink model used with QUARC to run the PV and PIV position controllers on the SRV02.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your SRV02 setup. If they have not been configured already, then you need to go to Section 2.4.3 to configure the lab files first.

1. Run the *setup_srv02_exp02_pos.m* script.
2. Enter the proportional and velocity control gains found in Pre-Lab question 4.
3. Set Signal Type in the SRV02 Signal Generator to *square* to generate a step reference.
4. Set the *Amplitude (rad)* gain block to $\pi/8$ to generate a step with an amplitude of 45 degrees.
5. Open the load shaft position scope, *theta_l (rad)*, and the motor input voltage scope, *Vm (V)*.
6. Click on QUARC | Build to compile the Simulink®diagram.
7. Select QUARC | Start to begin running the controller. The scopes should display responses similar to figures 2.13 and 2.14 Note that in the theta_l (rad) scope, the yellow trace is the setpoint position while the purple trace is the measured position.
8. When a suitable response is obtained, click on the Stop button in the Simulink® diagram toolbar

   (or select QUARC | Stop from the menu) to stop running the code. Generate a Matlab®figure showing the PV position response and its input voltage.

   As in the *s_srv02_pos* Simulink diagram, when the controller is stopped each scope automatically saves their response to a variable in the Matlab®workspace. Thus the *theta_l (rad)* scope saves its response to the *data_pos* variable and the *Vm (V)* scope saves its data to the *data_vm* variable.

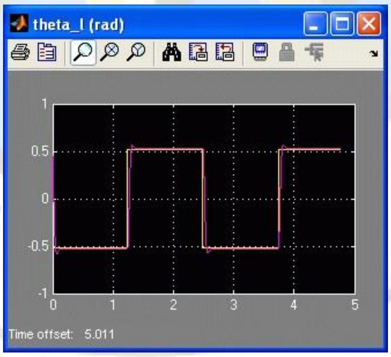

Figure 2.13: Measured PV step response.

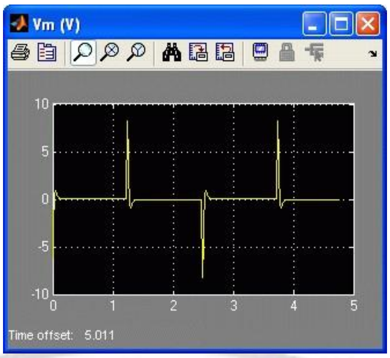

Figure 2.14: PV control input voltage.

If the experimental procedure is followed correctly, the measured SRV02 closed-loop position step response with the PV controller should be similar to Figure 2.15.

Answer 2.3.5

To generate this response, execute the sample_meas_tp_os.m script with the saved MAT files data_step_rsp_theta.mat and data_step_rsp_Vm.mat. Alternatively, to generate a Matlab^r figure from a new experimental run do the following:

(a) Execute the *setup_srv02_exp02_pos.m* script with CONTROL_TYPE = 'AUTO_PV'

(b) Run the *q_srv02_pos* Simulink^r model until a response fills the scopes.

(c) Stop QUARC^r.

(d) Execute the *sample_meas_tp_os.m* script.

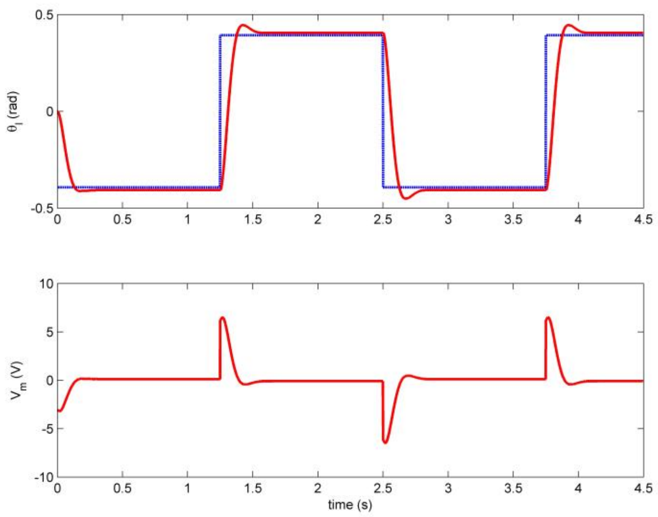

Figure 2.15: Measured SRV02 step response using PV.

9. Measure the steady-state error, the percent overshoot, and the peak time of the SRV02 load gear. Does the response satisfy the specifications given in Section 2.1.1.3?

Answer 2.3.6

Outcome Solution

K-1 The steady-state error measured in Figure 2.15 at 1.1 second after the peak time is

$$e_{ss} = 0.0138 \text{ rad} \quad \text{(Ans.2.3.7)}$$

Thus, there is an error of about 0.79 degrees. The peak time and percent overshoot of the response shown in Figure 2.15, using Equation 2.1.7 and Equation 2.1.6, are

$$t_p = 0.147 \text{ s} \quad \text{(Ans.2.3.8)}$$

and

$$PO = 4.88 \% \quad \text{(Ans.2.3.9)}$$

B-9 The actual measured SRV02 response does not quite satisfy the specifications given in Section 2.1.1.3 because the steady-state error is not zero. However, without saturating the servo motor the peak time does not exceed 0.20 seconds and the percent overshoot is below or equal to 5 %. Thus, the peak time and overshoot specifications are satisfied. The system, generally speaking, is more damped than predicted which leads to a lower overshoot. The constant steady-state error obtained along with the lower overshoot is due to un-modeled effects, notably friction. To find the steadystate error, peak time and percent overshoot of a response saved in *data_pos* automatically, run the *sample_meas_tp_os.m* script after running *q_srv02_pos* or using the responses saved in the MAT files *data_step_rsp_theta.mat* and *data_step_rsp_Vm.mat*.

10. Click the Stop button on the Simulink^r diagram toolbar (or select QUARC | Stop from the menu) to stop the experiment.
11. Turn off the power to the amplifier if no more experiments will be performed on the SRV02 in this session.

2.3.2 Ramp Response Using PV Controller

2.3.2.1 Simulation

In this simulation, the goal is to verify that the system with the PV controller can meet the zero steady-state error specification without saturating the motor.

As in the Step Response experiment in Section 2.3.1, in this experiment you need to use the *s_srv02_pos* Simulink® diagram shown in Figure 2.7 in Section 2.3.1.1 again.

1. Enter the proportional and velocity control gains found in Pre-Lab question 4.
2. Set the *SRV02 Signal Generator* parameters to the following to generate a triangular reference (which corresponds to a ramp input):
    - Signal Type = triangle
    - Amplitude = 1
    - Frequency = 0.8 Hz
3. Setting the frequency to 0.8 Hz will generate an increasing and decreasing ramp signal with the same slope used in the Pre-Lab question 6. The slope is calculated from the *Triangular Waveform* amplitude, *Amp*, and frequency, *f*, using the expression.

$$R_0 = 4Ampf \tag{2.3.36}$$

4. In the Simulink® diagram, set the *Amplitude (rad)* gain block to $\pi/3$.
5. Inside the *PIV Control* subsystem, set the *Manual Switch* to the down position so that the *High-Pass Filter* block is used.
6. Open the load shaft position scope, *theta_l (rad)*, and the motor input voltage scope, *Vm (V)*.
7. Start the simulation. The scopes should display responses similar to figures 2.16 and 2.17.

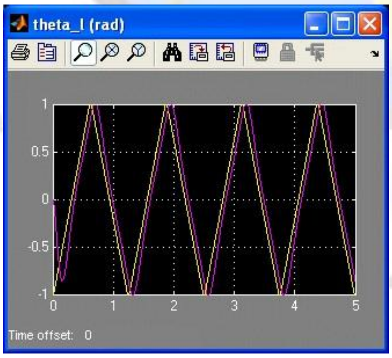

Figure 2.16: Ramp response using PV.

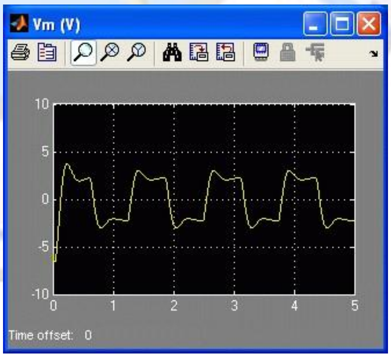

Figure 2.17: Input voltage of ramp tracking using PV.

8. Generate a Matlab® figure showing the *Ramp PV* position response and its corresponding input voltage trace.

Answer 2.3.7

Outcome Solution

K-3 The closed-loop ramp response when using the PV control is shown in Figure 2.18. This is generated using the *sample_meas_ess.m* script. To use this script, do the following:

(a) Execute the *setup_srv02_exp02_pos.m* script with CONTROL_TYPE = 'AUTO_PV' (b) Run the *s_srv02_pos* Simulink model.

(c) Run the *sample_meas_ess.m* script.

9. Measure the steady-state error. Compare the simulation measurement with the steady-state error calculated in Pre-Lab question 6.

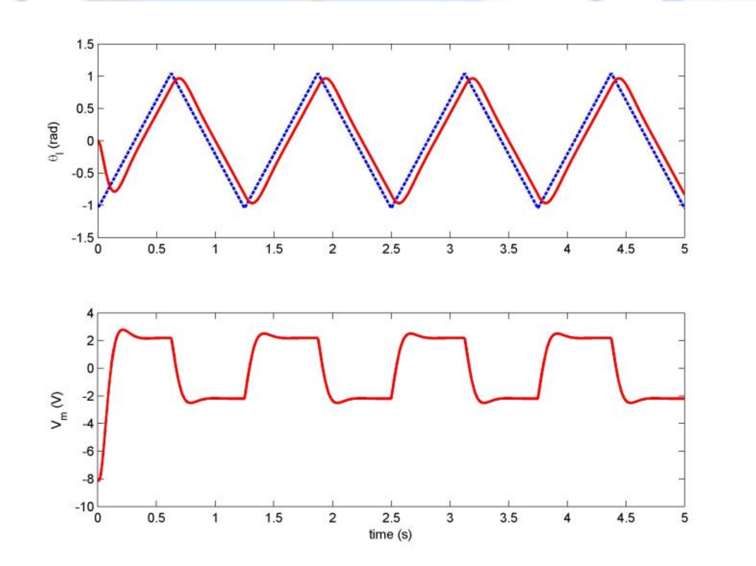

Figure 2.18: Ramp response using PV.

Answer 2.3.8

Outcome Solution

K-1 The error between the reference and the simulated response after running for 1.1 second is

$$e_{ss} = -0.213 \text{ rad} \qquad \text{(Ans.2.3.10)}$$

Its magnitude (or absolute value) is very close to the steady-state error predicted earlier in Ans.2.2.17. To find the steady-state error of a ramp response saved in *data_pos* automatically, run the *sample_meas_ess.m* script after running *s_srv02_pos*. It outputs the expected steady-state error when using the PV control, as found in the pre-lab, and the the error measured from the saved response.

2.3.2.2 Implementing Ramp Response Using PV

In this experiment, we will control the angular position of the SRV02 load shaft, i.e. the disc load, using a PV controller. The goal is to examine how well the system can track a triangular (ramp) position input. Measurements will then be taken to ensure that the specifications are satisfied.

As in the Step Response experiment in Section 2.3.1, in this experiment you also need to use the *q_srv02_pos* Simulink^{r}diagram shown in Figure 2.12 to implement the position control experiments.

1. Run the *setup_srv02_exp02_pos.m* script.
2. Enter the proportional and velocity control gains found in Pre-Lab question 4.
3. Set the *SRV02 Signal Generator* parameters to the following to generate a triangular reference (i.e., ramp reference):
    - Signal Type = triangle
    - Amplitude = 1
    - Frequency = 0.8 Hz
4. In the Simulink^{r}diagra, set the *Amplitude (rad)* gain block to $\pi/3$.
5. Open the load shaft position scope, *theta_l (rad)*, and the motor input voltage scope, *Vm (V)*.
6. Click on QUARC | Build to compile the Simulink^{r}diagram.
7. Select QUARC | Start to run the controller. The scopes should display responses similar to figures 2.19 and
   2.20.

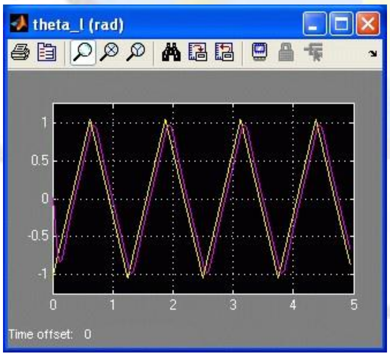

Figure 2.19: Measured SRV02 PV ramp response.

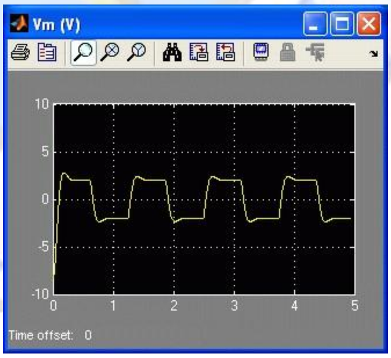

Figure 2.20: Input voltage of PV ramp response.

8. Generate a Matlab^{r}figure showing the *Ramp PV* position response and its corresponding input voltage trace.

Answer 2.3.9

Outcome Solution

B-5 If the experimental procedure is followed correctly, the measured SRV02 closed-loop position ramp response when using the PV control should be similar to Figure 2.21.

K-3 To generate this response, execute the sample_meas_ess_os.m script with the saved MAT files data_step_rsp_pv_theta.mat and data_step_rsp_pv_Vm.mat. Alternatively, to generate a Matlab[r] figure from a new experimental run do the following:

(a) Execute the *setup_srv02_exp02_pos.m* script with CONTROL_TYPE = 'AUTO_PV'

(b) Run the *q_srv02_pos* Simulink[r] model until a response fills the scopes.

(c) Stop QUARC[r].

(d) Execute the *sample_meas_ess_.m* script.

□□□

9. Measure the steady-state error and compare it with the steady-state error calculated in Pre-Lab question 6.

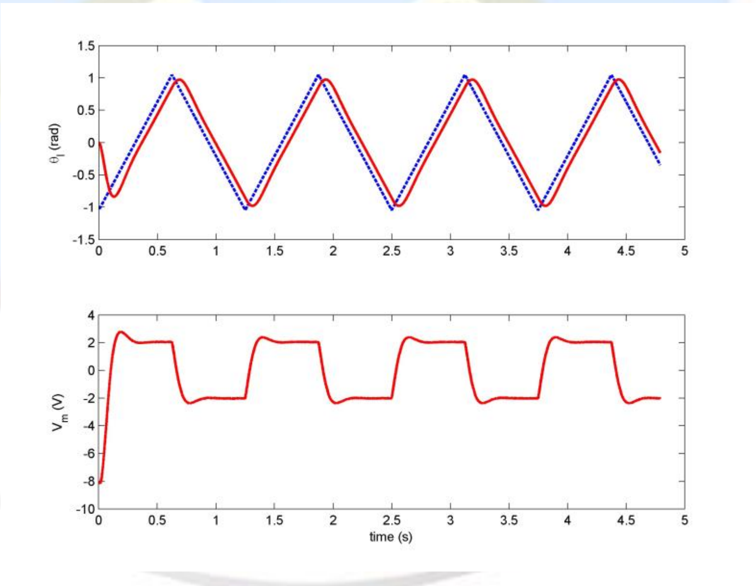

Figure 2.21: Measured SRV02 ramp response using PV.

Answer 2.3.10

Outcome Solution

K-1 The error between the reference and the measured response taken at the 1.0 second mark is

$e_{ss} = 0.189$ rad (Ans.2.3.11)

B-9 This is slightly less than the steady-state error calculated earlier in Ans.2.2.17.
To find the steady-state error of a ramp response saved in *data_pos automatically*, run the *sample_meas_ess.m* script using the saved response in the MAT files *data_step_rsp_pv_theta.mat* and *data_step_rsp_pv_Vm.mat* or after running *q_srv02_pos*.

2.3.3 Ramp Response with No Steady-State Error

Design an experiment to see if the steady-state error can be eliminated when tracking a ramp input. First simulate the response, then implement it using the SRV02 system.

1. How can the PV controller be modified to eliminate the steady-state error in the ramp response? State your hypothesis and describe the anticipated cause-and-effect leading to the expected result. **Hint:** Look through Section [1].

Answer 2.3.11

Outcome Solution

B-1 Hypothesis: Adding an integral control will eliminate the steady-state error. Because, the integrator will accumulate the error over time causing the input voltage to the motor to increase to make up for the additional voltage needed to eliminate the steady-state error.

Answer 2.3.12

Outcome Solution

B-2 Refering to the controller in Figure 2.6, the dependent variable is $V_m(s)$ and the independent variables are k_p, k_v, k_i, $\theta_d(s)$ and $\theta_l(s)$.

3. Your proposed control, like the PV compensator, are model-based controllers. This means that the control gains generated are based on mathematical representation of the system. Given this, list the assumptions you are making in this control design. State the reasons for your assumptions.

Answer 2.3.13

Outcome Solution

B-3 We assume that the friction in the system is negligable because it is a well-designed system. Also, noise in the

---

measured signals is neglected since its magnitude is very small compared to the magnitude of the measured signals.

4. Give a brief, general overview of the steps involved in your experimental procedure for two cases: (1) Simulation, and (2) Implementation.

Answer 2.3.14

Outcome

Solution B-4

Simulation case:

(a) Enter the integral gain computed in Pre-Lab question 7 into Matlab^{r}(Simulink diagram given in Figure 2.7).

(b) Start the simulation. The *Ramp PIV* response in the scopes should be similar to figures 2.22 and 2.23.

Implementation case:

(a) Enter the integral gain computed in Pre-Lab question 7 into Matlab^{r}(Simulink diagram given in Figure 2.12. Start the QUARC controller. The *Ramp PIV* response in the scopes should be similar to figures 2.24 and 2.25.

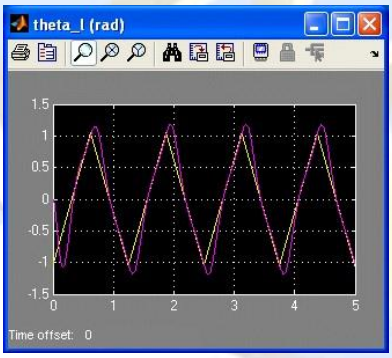

Figure 2.22: PIV ramp response.

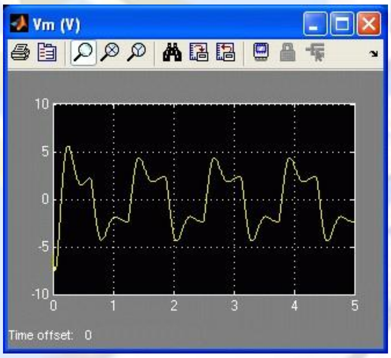

Figure 2.23: Input voltage using PIV control.

5. For each case, generate a Matlab^{r}figure showing the position response of the system and its corresponding input voltage.

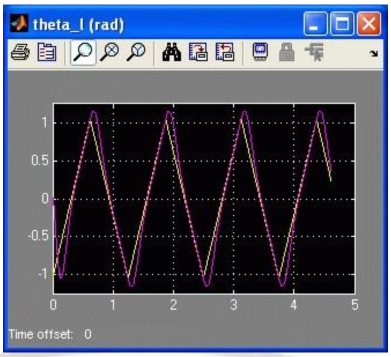

Figure 2.24: Measured SRV02 PIV ramp response.

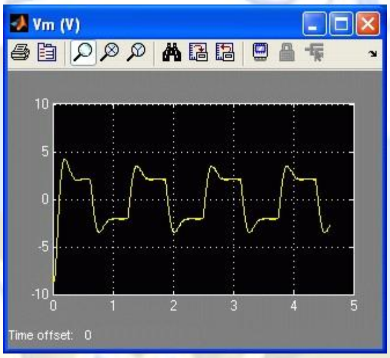

Figure 2.25: Input voltage from PIV ramp control.

K-3 **Simulation case:** After setting the integral gain in Ans.2.3.3 in Matlab^r^, the *PIV Ramp* response will be as shown in Figure 2.26. This plot can be generated by:

(a) Executing the *setup_srv02_exp02_pos.m* script with CONTROL_TYPE = 'AUTO_PV'

(b) Running the *s_srv02_pos* Simulink model.

(c) Running the *sample_meas_ess.m* script.

K-3 **Implementation case:** To generate this response, run the *sample_meas_ess.m* script using the saved response in the MAT files *data_step_rsp_piv_theta.mat* and *data_step_rsp_piv_Vm.mat*. To generate a response after running q_srv02_pos, follow these steps:

(a) Execute the *setup_srv02_exp02_pos.m* script with *CONTROL_TYPE = 'AUTO_PIV'*

(b) Run the *q_srv02_pos* Simulink^r^model.

(c) Stop QUARC^r^when a response fills the scopes.

(d) Run the *sample_meas_ess.m* script.

6.In each case, measure the steady-state error.

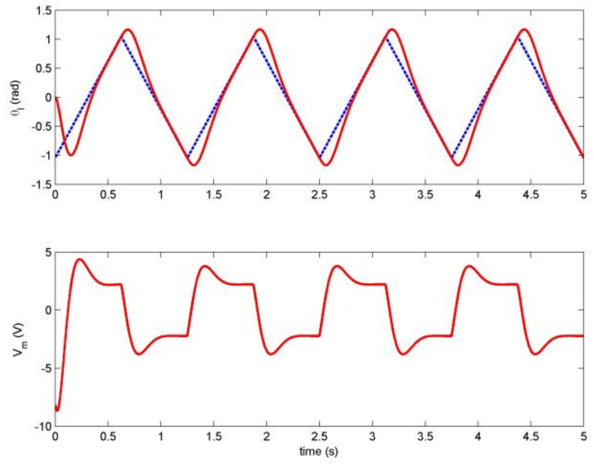

Figure 2.26: Ramp response using PIV control.

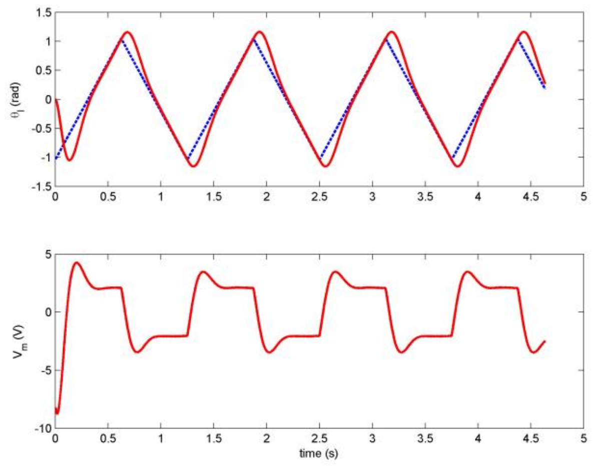

Figure 2.27: Measured SRV02 closed-loop ramp response using PIV.

Answer 2.3.16

Outcome Solution

K-1 **Simulation case:** The steady-state PIV error measured from the response in Figure 2.26 is

$$e_{ss} = -0.0125[rad] \qquad \text{(Ans.2.3.12)}$$

and the input voltage is always below 10.0 V. This value is reasonably close to 0.00 rad. Therefore, the specification is satisfied. To measure the steady-state error saved in *data_pos* automatically, run the *sample_meas_ess.m* script after running *s_srv02_pos*.

K-1 **Implementation case:** The steady-state PIV error measured from the response in Figure 2.27 at the 1.0 second mark is

$$e_{ss} = -0.0343 \text{ rad} \qquad \text{(Ans.2.3.13)}$$

□□□

7. For each case comment on whether the steady-state specification given in Section 2.1.1.3 was satisfied without saturating the actuator.

Answer 2.3.17

Outcome	Solution
B-9	**Both cases:** Given that the servo motor is not saturated and the obtained error is reasonably close to 0.00 rad, the specification are satisfied in both cases.

8. Click the Stop button on the Simulink[r] diagram toolbar (or select QUARC | Stop from the menu) to stop the experiment.

9. Turn off the power to the amplifier if no more experiments will be performed on the SRV02 in this session.

2.3.4 Results

Fill out Table 2.2 below with your answers to the Pre-Lab questions and your results from the lab experiments.

Section / Question	Description	Symbol	Value	Unit
Question 4	**Pre-Lab: Model Parameters**			
	Open-Loop Steady-State Gain	K	1.53	rad/(V.s)
	Open-Loop Time Constant	τ	0.0254	s
Question 4	**Pre-Lab: PV Gain Design**			
	Proportional gain	k_p	7.82	V/rad
	Velocity gain	k_v	-0.157	V.s/rad
Question 5	**Pre-Lab: Control Gain Limits**			
	Maximum proportional gain	$k_{p,max}$	12.7	V/rad
Question 6	**Pre-Lab: Ramp Steady-State Error**			
	Steady-state error using PV	e_{ss}	0.214	rad
Question 7	**Pre-Lab: Integral Gain Design**			
	Integral gain	k_i	38.9	V/(rad.s)
2.3.1.1	**Step Response Simulation**			
	Peak time	t_p	0.20	s
	Percent overshoot	PO	5.0	%
	Steady-state error	e_{ss}	0.00	rad
2.3.1.1	**Filtered Step Response Using PV**			
	Peak time	t_p	0.20	s
	Percent overshoot	PO	5.76	%
	Steady-state error	e_{ss}	0.00	rad
2.3.1.2	**Step Response Implementation**			
	Peak time	t_p	0.147	s

	Percent overshoot Steady-state error	PO e_{ss}	4.88 0.0138	% rad
2.3.2.1	**Ramp Response Simulation with PV** Steady-state error	e_{ss}	-0.213	rad
2.3.2.2	**Ramp Response Implementation with PV** Steady-state error	e_{ss}	0.189	rad
2.3.3	**Ramp Response Simulation with no steady-state error** Steady-state error	e_{ss}	-0.0125	rad
2.3.3	**Ramp Response Implementation with no steady-state error** Steady-state error	e_{ss}	-0.0343	rad

Table 2.2: Summary of results for the SRV02 Position Control laboratory.

2.4 System Requirements

Before you begin this laboratory make sure:

- QUARC^r is installed on your PC, as described in Reference [1].
- You have a QUARC compatible data-aquisition (DAQ) card installed in your PC. For a listing of compliant DAQ cards, see Reference [5].
- SRV02 and amplifier are connected to your DAQ board as described Reference [6].

2.4.1 Setup for Position Control Implementation

Before beginning the lab experiments on the SRV02 device, the q_srv02_pos Simulink^r diagram and the setup_srv02_exp02_pos script must be configured.

Follow these steps to get the system ready for this lab:

1. Setup the SRV02 in the high-gear configuration and with the disc load as described in Reference [6].
2. Load the Matlab^r software.
3. Browse through the *Current Directory* window in Matlab^r and find the folder that contains the SRV02 position control files, e.g. *q_srv02_pos.mdl*.
4. Double-click on the *q_srv02_pos.mdl* file to open the Position Control Simulink^r diagram shown in Figure 2.7.

5. **Configure DAQ:** Double-click on the HIL Initialize block in the *SRV02-ET* subsystem (which is located inside the *SRV02-ET* Position subsystem) and ensure it is configured for the DAQ device that is installed in your system. See Section A for more information on configuring the HIL Initialize block.

6. **Configure Sensor:** The position of the load shaft can be measured using various sensors. Set the Pos Src Source block in q_srv02_pos, as shown in Figure 2.7, as follows:

   - 1 to use the potentiometer
   - 2 to use to the encoder

   Note that when using the potentiometer, there will be a discontinuity.

7. Configure setup script: Set the parameters in the setup_srv02_exp02_pos.m script according to your system setup. See Section 2.4.2 for more details.

SRV02 SPEED CONTROL

The objective of this laboratory is to develop feedback systems that control the speed of the rotary servo load shaft. A proportional-integral (PI) controller and a lead compensator are designed to regulate the shaft speed according to a set of specifications.

Topics Covered

- Design of a proportional-integral (PI) controller that regulates the angular speed of the servo load shaft.
- Design of a lead compensator.
- Simulation of the PI and lead controllers using the plant model to ensure the specifications are met without any actuator saturation.
- Implemention of the controllers on the Quanser SRV02 device to evaluate their performance.

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- Data acquisition device (e.g. Q2-USB), the power amplifier (e.g. VoltPAQ-X1), and the main components of the SRV02 (e.g. actuator, sensors), as described in References [2], [4], and [6], respectively.

- Wiring and operating procedure of the SRV02 plant with the amplifier and data-aquisition (DAQ) device, as discussed in Reference [6].
- Transfer function fundamentals, e.g. obtaining a transfer function from a differential equation.
- Laboratory described in Appendix A to get familiar with using QUARC[r]with the SRV02.

3.1 Background

3.1.1 Desired Response

3.1.1.1 SRV02 Speed Control Specifications

The time-domain requirements for controlling the speed of the SRV02 load shaft are:

$$e_{ss} = 0 \tag{3.1.1}$$

$$t_p \leq 0.05 \text{ s, and} \tag{3.1.2}$$

$$PO \leq 5\ \% \tag{3.1.3}$$

Thus, when tracking the load shaft reference, the transient response should have a peak time less than or equal to 0.05 seconds, an overshoot less than or equal to 5 %, and zero steady-state error.

In addition to the above time-based specifications, the following frequency-domain requirements are to be met when designing the *Lead Compensator*:

$$PM \geq 75.0 \text{ deg} \tag{3.1.4}$$

and

$$\omega_g = 75.0 \text{ rad/s} \tag{3.1.5}$$

The phase margin mainly affects the shape of the response. Having a higher phase margin implies that the system is more stable and the corresponding time response will have less overshoot. The overshoot will not go beyond 5% with a phase margin of at least 75.0 degrees.

The crossover frequency is the frequency where the gain of the Bode plot is 1 (or 0 dB). This parameter mainly affects the speed of the response, thus having a larger ω_g decreases the peak time. With a crossover frequency of 75.0 radians the resulting peak time will be less than or equal to 0.05 seconds.

3.1.1.2 Overshoot

In this laboratory we will use the following step setpoint (input):

$$\omega_d(t) = \begin{cases} 2.5 \text{ rad/s} & t \leq t_0 \\ 7.5 \text{ rad/s} & t > t_0 \end{cases} \quad (3.1.6)$$

where t_0 is the time the step is applied. Initially, the SRV02 should be running at 2.5 rad/s and after the step time it should jump up to 7.5 rad/s. From the standard definition of overshoot in step response, we can calculate the maximum overshoot of the response (in radians):

$$\omega(t_p) = \omega_d(t_0) + (\omega_d(t) - \omega_d(t_0))\left(1 + \frac{PO}{100}\right) \quad (3.1.7)$$

with the given values the maximum overshoot of the response is

$$\omega(t_p) = 7.75 \text{ rad/s} \quad (3.1.8)$$

The closed-loop speed response should therefore not exceed the value given in Equation 3.1.8.

3.1.1.3 Steady State Error

Consider the speed control system with unity feedback shown in Figure 3.1. Let the compensator be $C(s) = 1$.

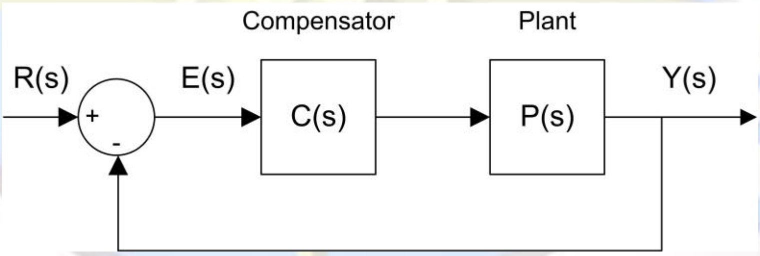

Figure 3.1: Unity feedback loop.

We can find the steady-state error using the final value theorem:

$$e_{ss} = \lim_{s \to 0} sE(s) \quad (3.1.9)$$

where

$$E(s) = \frac{R(s)}{1 + C(s)\,P(s)} \quad (3.1.10)$$

The voltage-to-speed transfer function for the SRV02 was found in Section 1 as:

$$P(s) = \frac{K}{\tau\, s + 1} \quad (3.1.11)$$

Substituting $R(s) = \frac{R_0}{s}$ and $C(s) = 1$ gives:

$$E(s) = \frac{R_0}{s\left(1 + \frac{K}{\tau\, s+1}\right)} \quad (3.1.12)$$

Applying the final-value theorem to the system gives

$$e_{ss} = R_0 \left(\lim_{s\to 0} \frac{\tau\, s+1}{\tau\, s+1+K}\right) \quad (3.1.13)$$

When evaluated, the resulting steady-state error due to a step response is

$$e_{ss} = \frac{R_0}{1+K} \quad (3.1.14)$$

3.1.2 PI Control Design

3.1.2.1 Closed Loop Transfer Function

The proportional-integral (PI) compensator used to control the velocity of the SRV02 has the following structure:

$$V_m(t) = k_p\,(b_{sp}\,\omega_d(t) - \omega_l(t)) - k_i \int (\omega_d(t) - \omega_l(t))dt \quad (3.1.15)$$

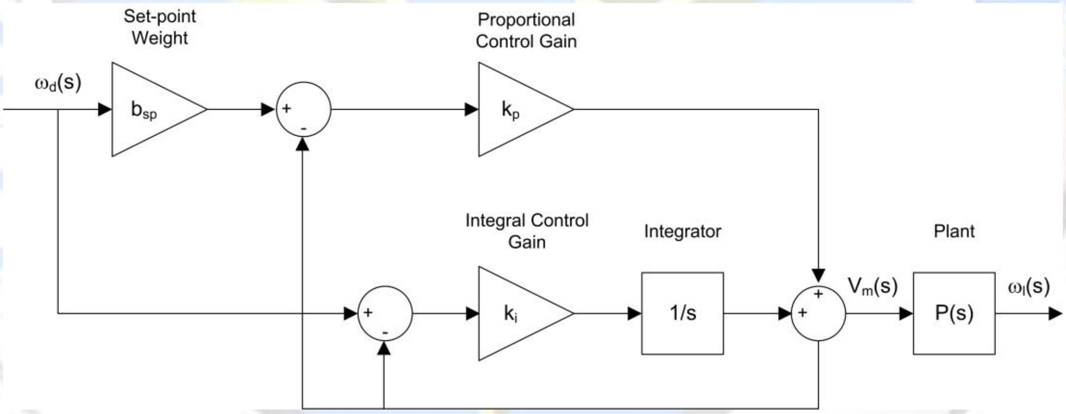

Figure 3.2: Block diagram of SRV02 PI speed control.

where k_p is the proportional control gain, k_i is the integral control gain, $\omega_d(t)$ is the setpoint or reference angular speed for the load shaft, $\omega_l(t)$ is the measured load shaft angular speed, b_{sp} is the setpoint weight, and $V_m(t)$ is the voltage applied to the SRV02 motor. The block diagram of the PI control is given in Figure 3.2.

We can take Laplace transform of the controller given in Equation 3.1.15:

$$V_m(s) = k_p\ (b_{sp}\,\Omega_d(s) - \Omega_l(s)) + \frac{k_i\ (\Omega_d(s) - \Omega_l(s))}{s} \quad (3.1.16)$$

To find the closed-loop speed transfer function, $\Omega_l(s)/\Omega_d(s)$, we can use the process transfer function from Equation 3.1.11 and solve for $\Omega_l(s)/\Omega_d(s)$ as:

$$\frac{\Omega_l(s)}{\Omega_d(s)} = \frac{K\ (k_p\, s\, b_{sp} + k_i)}{s^2\, \tau + (1 + K\, k_p)\ s + K\, k_i} \qquad (3.1.17)$$

3.1.2.2 Finding PI Gains to Satisfy Specifications

In this section, we will first calculate the minimum damping ratio and natural frequency required to meet the specifications given in Section 3.1.1.1. Then, using these values we will calculate the necessary control gains k_p and k_i to achieve the desired performance with a PI controller.

The minimum damping ratio and natural frequency needed to satisfy a given percent overshoot and peak time are:

$$\zeta = -\ln\left(\frac{PO}{100}\right)\sqrt{\frac{1}{\ln\left(\frac{PO}{100}\right)^2 + \pi^2}} \qquad (3.1.18)$$

and

$$\omega_n = \frac{\pi}{t_p\, \sqrt{1 \quad \zeta^2}} \qquad (3.1.19)$$

–

Substituting the percent overshoot specifications given in 3.1.3 into Equation 3.1.18 gives the required damping ratio

$$\zeta = 0.690 \qquad (3.1.20)$$

Then, by substituting this damping ratio and the desired peak time, given in 3.1.2, into Equation 3.1.19, the minimum natural frequency is found as:

$$\omega_n = 86.7 \text{ rad/s} \qquad (3.1.21)$$

Now, let's look at how we can calculate the gains. When the setpoint weight is zero, i.e. $b_{sp} = 0$, the closed-loop SRV02 speed transfer function has the structure of a *standard second-order system*. We can find expressions for the control gains k_p and k_i by equating the characteristic equation (denominator) of the SRV02 closed-loop transfer function to the *standard characteristic equation*: $s^2 + 2\zeta\, \omega_n\, s + \omega_n^2$.

The denominator of the transfer function can be re-structured into the following:

$$s^2 + \frac{(1 + K\, k_p)\ s}{\tau} + \frac{K\, k_i}{\tau} \qquad (3.1.22)$$

equating the coefficients of this equation to the coefficients of the standard characteristic equation gives:

$$\frac{K\, k_i}{\tau} = \omega_n^2 \qquad (3.1.23)$$

and

$$\frac{1 + K\, k_p}{\tau} = 2\,\zeta\,\omega_n \qquad (3.1.24)$$

Then, the proportional gain k_p can be found as:

$$k_p = \frac{-1 + 2\,\zeta\,\omega_n\,\tau}{K} \qquad (3.1.25)$$

and the integral gain k_i is

$$k_i = \frac{\omega_n^2\,\tau}{K} \tag{3.1.26}$$

3.1.3 Lead Control Design

Alternatively, a lead or lag compensator can be designed to control the speed of the servo. The lag compensator is actually an approximation of a PI control and this, at first, may seem like the more viable option. However, due to the saturation limits of the actuator the lag compensator cannot achieve the desired zero steady-state error specification. Instead, a lead compensator with an integrator, as shown in Figure 3.3, will be designed.

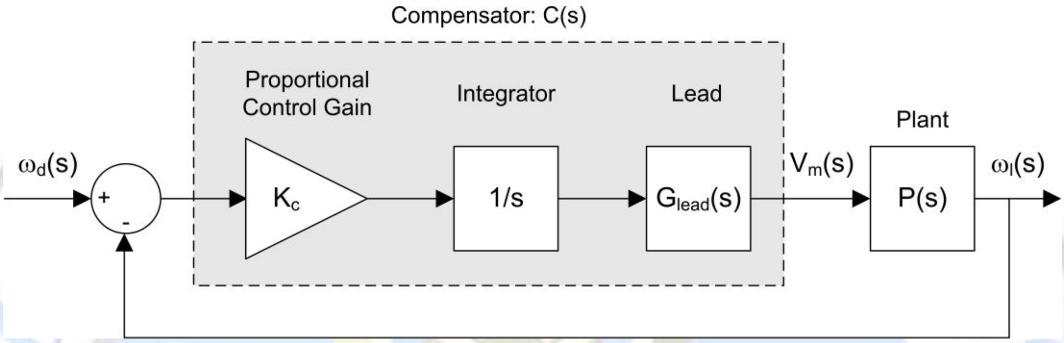

Figure 3.3: Closed-loop SRV02 speed control with lead compensator.

To obtain zero steady-state error, an integrator is placed in series with the plant. This system is denoted by the transfer function

$$P_i(s) = \frac{P(s)}{s} \tag{3.1.27}$$

where $P(s)$ is the plant transfer function in Equation 3.1.11.

The phase margin and crossover frequency specifications listed in equations 3.1.4 and 3.1.5 of Section 3.1.1.1 can then be satisfied using a proportional gain K_c and the lead transfer function

$$G_{lead}(s) = \frac{1 + a\,T\,s}{1 + T\,s} \tag{3.1.28}$$

The a and T parameters change the location of the pole and the zero of the lead compensator which changes the gain and phase margins of the system. The design process involves examining the stability margins of the *loop transfer function*, $L(s) = C(s) \cdot P(s)$, where the compensator is given by:

$$C(s) = \frac{K_c\,(1 + a\,T\,s)}{(1 + T\,s)\,s} \tag{3.1.29}$$

3.1.3.1 Finding Lead Compensator Parameters

The Lead compensator is an approximation of a proportional-derivative (PD) control. A PD controller can be used to add damping to reduce the overshoot in the transient of a step response and effectively making the system more stable. In other words, it increases the phase margin. In this particular case, the lead compensator is designed for the following system:

$$L_p(s) = \frac{K_c\,P(s)}{s} \tag{3.1.30}$$

The proportional gain Kc is designed to attain a certain crossover frequency. Increasing the gain crossover frequency essentially increases the bandwidth of the system which decreases the peak time in the transient response (i.e. makes the response faster). However, as will be shown, adding a gain $K_c > 1$ makes the system less stable. The phase margin of the $L_p(s)$ system is therefore lower than the phase margin of the $P_i(s)$ system and this translates to having a large overshoot in the response. The lead compensator is used to dampen the overshoot and increase the overall stability of the system, i.e increase its phase margin.

The frequency response of the lead compensator given in 3.1.28 is

$$G_{lead}\,(\omega\,j) = \frac{1 + a\,T\,\omega j}{1 + T\,\omega\,j} \tag{3.1.31}$$

and its corresponding magnitude and phase equations are

$$|G_{lead}(\omega\,j)| = \sqrt{\frac{T^2\,\omega^2\,a^2 + 1}{1 + T^2\,\omega^2}} \tag{3.1.32}$$

and

$$\phi_G = arctan(a\,T\omega) - arctan(T\,\omega) \tag{3.1.33}$$

The Bode plot of the lead compensator is shown in Figure 3.4.

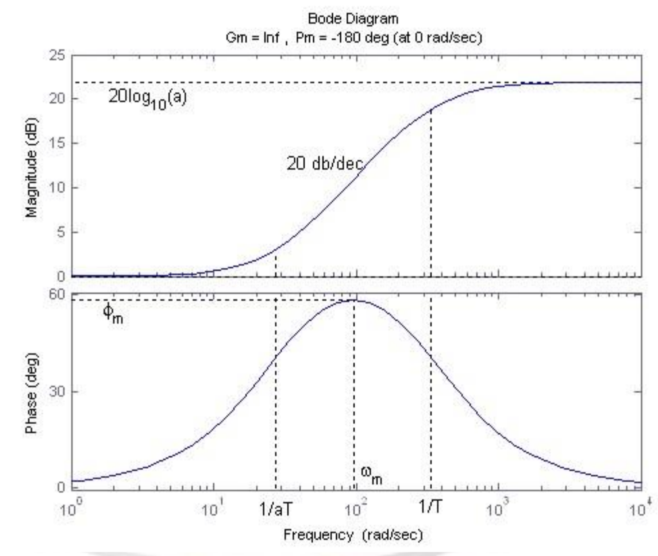

Figure 3.4: Bode of lead compensator.

3.1.3.2 Lead Compensator Design using MATLAB

In this section, we will use Matlab[r] to design a lead compensator that will satisfy the frequency-based specifications given in Section 3.1.1.1.

1. **Bode plot of the open-loop uncompensated system**, $P_i(s)$, must first be found. To generate the Bode plot of $P_i(s)$, enter the following commands in Matlab[r]. **NOTE:** If your system has not been set up yet, then you need to first run the the *setup_srv02_exp03_spd.m* script. This script will store the model parameter *K* and *tau* in the Matlab[r]workspace. These parameters are used with the commands *tf* and *series* to create the $P_i(s)$ transfer function. The *margin* command generates a Bode plot of the system and it lists the gain and phase stability margins as well as the phase and gain crossover frequencies.

```
% Plant transfer function
P = tf([K],[tau 1]);
% Integrator transfer function
I = tf([1],[1 0]);
% Plant with Integrator transfer function
Pi =
series(P,I)
; % Bode of
Pi(s)
figure(1)
margin(Pi);
set
(1,'name','
Pi(s)');
```

The entire Lead compensator design is given in the *d_lead.m* script file. Run this script after running the setup_srv02_exp03_spd.m script when CONTROL_TYPE = 'AUTO' to generate a collection of Bode diagrams including the Bode of $P_i(s)$ given in Figure 3.5.

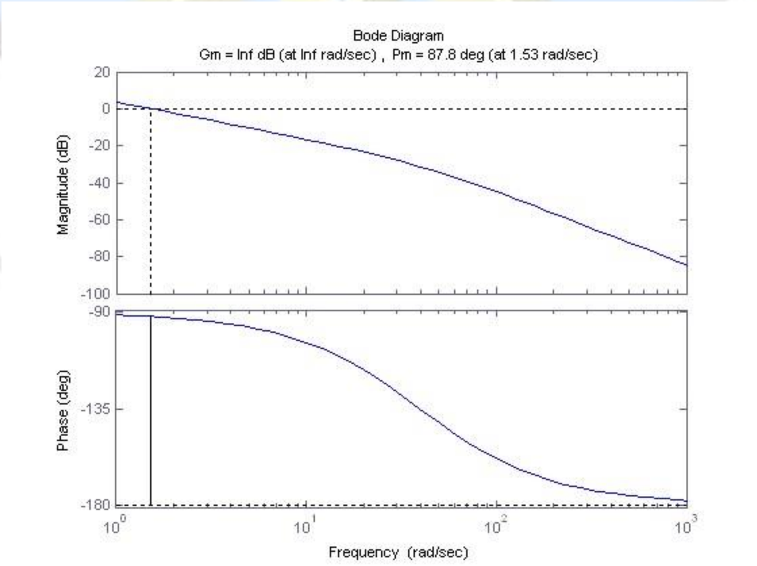

Figure 3.5: Bode of Pi(s) system.

2. **Find how much more gain is required** such that the gain crossover frequency is 50.0 rad/s (use the *ginput* Matlab^{r}command). As mentioned before, the lead compensator adds gain to the system and will increases the phase as well. Therefore, gain K_c is not to be designed to meet the specified 75.0 rad/s fully.

   As given in Figure 3.5, the crossover frequency of the uncompensated system is 1.53 rad/s. To move the crossover frequency to 50.0 rad/s, a gain of

$$K_c = 34.5 \text{ dB} \tag{3.1.34}$$

   or

$$K_c = 53.1 \text{ V/rad} \tag{3.1.35}$$

   in the linear range is required. The Bode plot of the loop transfer function $L_p(s)$ (from Section 3.1.3) is given in Figure 3.6. This initial estimate of the gain was found using the *ginput* command. The gain was then adjusted according to the crossover frequency calculated in the generated Bode plot of the $L_p(s)$ system. The commands used to generate the Bode plot are given in the d_lead.m script.

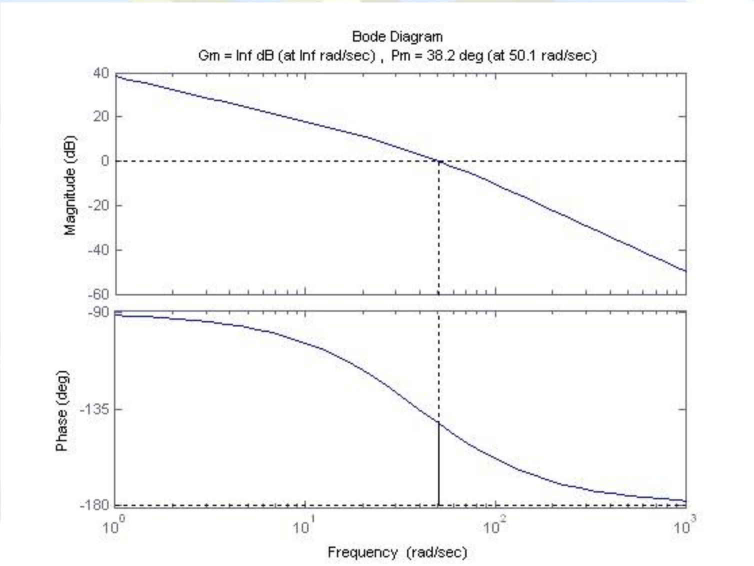

Figure 3.6: Bode of Lp(s) system.

3. **Gain needed for specified phase margin** must be found next so that the lead compensator can achieve the specified phase margin of 75 degrees. Also, to ensure the desired specifications are reached, we'll add another 5 degrees to the maximum phase of the lead.

   To attain the necessary phase margin, the maximum phase of the lead can be calculated using

$$\phi_m = PM_{des} - PM_{meas} + 5 \tag{3.1.36}$$

   Given that the desired phase margin in Equation 3.1.4 and the phase margin of $L_p(s)$ is

$$PM_{meas} = 21.5 \text{ deg} \tag{3.1.37}$$

the maximum lead phase has to be about

$$\phi_m = 41.8 \text{ deg} \tag{3.1.38}$$

or

$$\phi_m = 0.728 \text{ rad} \tag{3.1.39}$$

The lead compensator, as explained in Section 3.1.3.1, has two parameters: a and T . To attain the maximum phase ϕ_m shown in Figure 3.4, the Lead compensator has to add 20 $\log_{10}(a)$ of gain. This is determined using the equation

$$a = -\frac{1 + \sin(\phi_m)}{-1 + \sin(\phi_m)} \tag{3.1.40}$$

The gain needed is found by inserting the max phase into this equation to get

$$a = 4.96 \tag{3.1.41}$$

which is

$$20 \log 10(a) = 13.9 \text{ dB} \tag{3.1.42}$$

4. **The frequency at which the lead maximum phase occurs** must be placed at the new gain crossover frequency $\omega_{g,new}$. This is the crossover frequency after the lead compensator is applied. As illustrated in Figure 3.4, ω_m occurs halfway between 0 dB and $20 \log_{10}(a)$, i.e. at $10 \log_{10}(a)$. So, the new gain crossover frequency in the $L_p(s)$ system will be the frequency where the gain is $-10 \log_{10}(a)$.

   From Figure 3.6, it is found that the frequency where the $-10 \log_{10}(a)$ gain in the $L_p(s)$ system occurs is at about 80.9 rad/s. Thus, the maximum phase of the lead will be set to

$$\omega_m = 80.9 \text{ rad/s} \tag{3.1.43}$$

   As illustrated earlier in Figure 3.4 in Section 3.1.3.1, the maximum phase occurs at the maximum phase frequency ω_m. Parameter T given by:

$$T = \frac{1}{\omega_m \sqrt{a}} \tag{3.1.44}$$

   is used to attain a certain maximum phase frequency. This changes where the Lead compensator breakpoint frequencies $1/(a * T)$ and $1/T$ shown in Figure 3.4 occur. The slope of the lead compensator gain changes at these frequencies. We can find the parameter T by substituting $\omega_m = 80.9$ and the lead gain value from Equation 3.1.41 into Equation 3.1.44:

$$T = 0.00556 \tag{3.1.45}$$

   s/rad Therefore, the lead breakpoint frequencies are:

$$\frac{1}{aT} = 36.1 \text{ rad/s} \tag{3.1.46}$$

   and

$$\frac{1}{T} = 180.9 \text{ rad/s} \tag{3.1.47}$$

5. **Bode plot of the lead compensator** $C_{lead}(s)$, defined in 3.1.28 can be generated using the *d_lead.m* script.

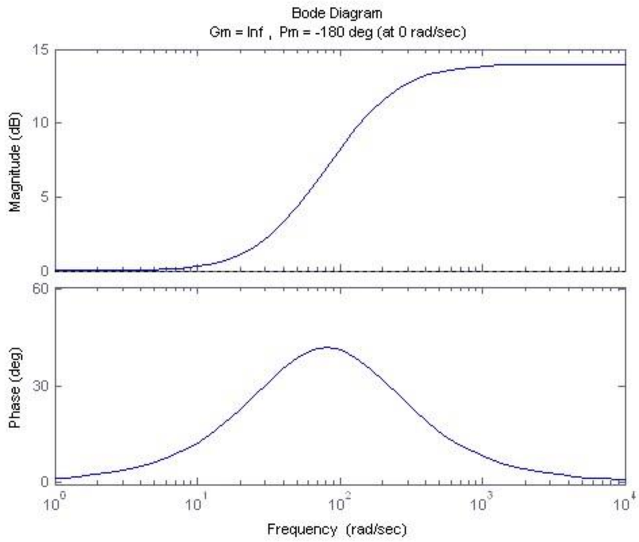

Figure 3.7: Bode of lead compensator $C_{lead}(s)$.

6. **Bode plot of the loop transfer function** $L(s)$, as described in 3.1.30, can be generated using the *d_lead.m* script. The phase margin of $L(s)$ is 68.1 degrees and is below the desired phase margin of 75.0 degrees, as specified in Section 3.1.1.
7. **Check response** by simulating the system to make sure that the time-domain specifications are met. Keep in mind that the goal of the lead design is the same as the PI control, the response should meet the desired steady-state error, peak time, and percentage overshoot specifications given in Section 3.1.1. Thus, if the crossover frequency and/or phase margin specifications are not quite satisfied, the response should be simulated to verify if the time-domain requirements are satisfied. If so, then the design is complete. If not, then the lead design needs to be re-visited.

   You will work on this later in the laboratory as described in Section 3.3.2.1.

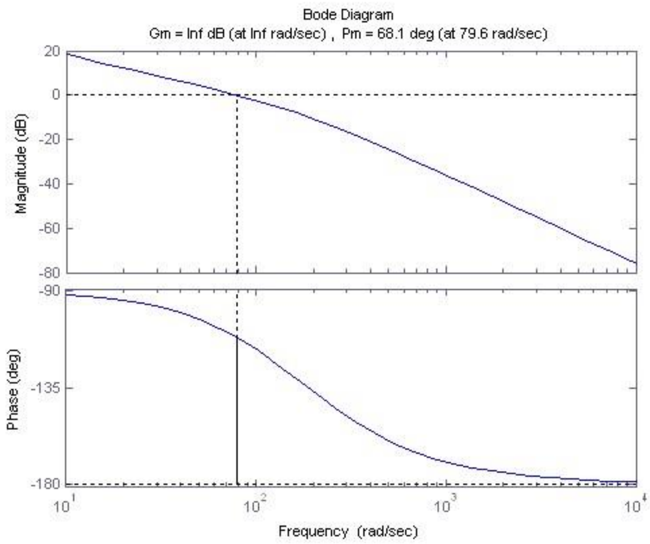

Figure 3.8: Bode of loop transfer function $L(s)$.

3.1.4 Sensor Noise

When using analog sensors, such as a tachometer, there is often some inherent noise in the measured signal.

The peak-to-peak noise of the measured SRV02 load gear signal can be calculated using

$$e_\omega = \frac{1}{100} K_n \omega_l \tag{3.1.48}$$

where K_n is the peak-to-peak ripple rating of the sensor and ω_l is the speed of SRV02 load gear. The rated peakto-peak noise of the SRV02 tachometer is given in Appendix B of Reference [6] as:

$$K_n = 7\ \% \tag{3.1.49}$$

Based on this specification, the peak-to-peak noise, when the load shaft runs at 7.5 rad/s, will be

$$e_\omega = 0.525 \text{ rad/s} \tag{3.1.50}$$

Thus, the signal will oscillate ± 0.2625 rad/s about the 7.5 rad/s setpoint, or approximately between 7.24 rad/s and 7.76 rad/s. Then, taking the noise into account, what would be the maximum peak in the speed response that is to be expected?

Equation 3.1.7 was used to find the peak value of the load gear response for a given percent overshoot. To take into account the noise in the signal, this formula is modified as follows:

$$\omega(t_p) = \omega_d(t_0) + (\omega_d(t) - \omega_d(t_0)) \left(1 + \frac{PO}{100}\right) + \frac{1}{2} e_\omega \tag{3.1.51}$$

Given a reference signal that goes between 2.5 rad/s to 7.5 rad/s, as described in Section 3.1.1.1, and the peak-topeak ripple estimate in Equation 3.1.50, the peak speed of the load gear, including the noise, can be found as:

$$\omega(t_p) = 8.01 \text{ rad/s} \tag{3.1.52}$$

Using

$$PO = \frac{100\ (\omega(t_p) - \omega_d(t))}{\omega_d(t) - \omega_d(t_0)} \tag{3.1.53}$$

the new maximum percent overshoot for a 5.0 rad/s step is

$$PO \leq 10.2\ \% \tag{3.1.54}$$

3.2 Pre-Lab Questions

1. Based on the steady-state error result of a step response from Equation ,what *type* of system is the SRV02 when performing speed control (Type 0, 1, or 2) and why?

Answer 3.2.1

Outcome Solution

A-3 This is a *Type 0* system because the steady-state error is a constant given a step reference.

2. The nominal SRV02 model parameters, K and τ, found in SRV02 Modeling Laboratory (Section 1) should be about 1.53 (rad/s-V) and 0.0254 sec, respectively. Calculate the PI control gains needed to satisfy the time-domain response requirements.

Answer 3.2.2

Outcome Solution

A-2 Using the nominal SRV02 model parameters

$$K = 1.53 \text{ rad/(V.s)} \quad \text{(Ans.3.2.1)}$$

and

$$\tau = 0.0254 \text{ s} \quad \text{(Ans.3.2.2)}$$

along with the damping ratio given in Equation 3.1.20 with Equation 3.1.25 generates the proportional control gain

$$k_p = 1.34 \text{ V/(rad/s)} \quad \text{(Ans.3.2.3)}$$

The integral control gain is obtained by substituting the model parameters given above with the minimum natural frequency specification given in 3.1.21 into Equation 3.1.26

$$k_i = 124.9 \text{ V/rad} \quad \text{(Ans.3.2.4)}$$

Thus, if these gains are used, the speed response of the load gear on an SRV02 with a disc load will satisfy the specifications listed in Section 3.1.1.1.

3.Find the frequency response magnitude, $|P_i(\omega)|$, of the transfer function $P_i(s)$ given in Equation 3.1.27.

Answer 3.2.3

Outcome Solution

A-2 The frequency response of $P_i(s)$ is found by substituting $s = j\,\omega$ in 3.1.27.

$$P_i\,(j\,\omega) = \frac{K}{(\tau\,j\,\omega + 1)\;j\,\omega}$$

(Ans.3.2.5)

Taking the magnitude of this expression gives the frequency response gain

$$|P_i(\omega)| = \frac{K}{\omega\sqrt{\tau^2\,\omega^2+1}} \qquad \text{(Ans.3.2.6)}$$

4. Calculate the DC gain of $P_i(s)$ given in Equation 3.1.27. **Hint:** The DC gain is the gain when the frequency is zero, i.e. $\omega = 0 rad/s$. However, because of its integrator, $P_i(s)$ has a singularity at zero frequency. Therefore, the DC gain is not technically defined for this system. Instead, approximate the DC gain by using $\omega = 1 rad/s$. Make sure the DC gain estimate is evaluated numerically in dB using the nominal model parameters, $K = 1.53$ and $\tau = 0.0254$, (or use what you found for K and τ in Section 1).

Answer 3.2.4

Outcome	Solution
A-2	Substituting $\omega = 1$ rad/s gives the approximate DC gain of

$$|P_i(1)| = \frac{K}{\sqrt{\tau^2+1}} \qquad \text{(Ans.3.2.7)}$$

Substituting the nominal SRV02 model parameters in the above expression results in the DC gain estimate of

$$|P_i(1)| = 1.53 \qquad \text{(Ans.3.2.8)}$$

or

$$|P_i(1)|_{dB} = 3.70 \text{ dB} \qquad \text{(Ans.3.2.9)}$$

5. The gain crossover frequency, ω_g, is the frequency at which the gain of the system is 1 or 0 dB. Express the crossover frequency symbolically in terms of the SRV02 model parameters K and τ. Then, evaluate the expression using the nominal SRV02 model parameters $K = 1.53$ and $\tau = 0.0254$, (or use what you found for K and τ in Section 1).

Answer 3.2.5

Outcome	Solution
A-1	The crossover frequency is found by setting $|P_i(\omega_g)| = 1$ in equation Ans.3.2.6 and solving for ω_g
A-2	

$$\omega_g = \frac{\sqrt{2}\sqrt{\frac{-1+\sqrt{1+4\,\tau^2\,K^2}}{\tau^2}}}{2} \qquad \text{(Ans.3.2.10)}$$

When evaluated with the nominal SRV02 parameters, the frequency where the gain is 0 dB is

$$\omega_g = 1.524 \text{ rad/s} \qquad \text{(Ans.3.2.11)}$$

3 SRV02 SPECIFICATIONS

Table 3.1 lists and characterizes the main parameters associated with the SRV02. Some of these are used in the mathematical model. More detailed information about the gears is given in Table 3.2 and the calibration gains for the various sensors on the SRV02 are summarized in Table 3.3.

Symbol	Description	Value	Variation
V_{nom}	Motor nominal input voltage	6.0 V	
R_m	Motor armature resistance	2.6 Ω	± 12%
L_m	Motor armature inductance	0.18 mH	
k_t	Motor current-torque constant	7.68×10^{-3} N-m/A	± 12%
k_m	Motor back-emf constant	7.68×10^{-3} V/(rad/s)	± 12%
K_g	High-gear total gear ratio	70	
	Low-gear total gear ratio	14	
η_m	Motor efficiency	0.69	± 5%
η_g	Gearbox efficiency	0.90	± 10%
$J_{m,rotor}$	Rotor moment of inertia	3.90×10^{-7} kg-m^2	± 10%
J_{tach}	Tachometer moment of inertia	7.06×10^{-8} kg-m^2	± 10%
J_{eq}	High-gear equivalent moment of inertia without external load	2.087×10^{-3} kg-m^2	
	Low-gear equivalent moment of inertia without external load	9.7585×10^{-5} kg-m^2	
B_{eq}	High-gear Equivalent viscous damping coefficient	0.015 N-m/(rad/s)	
	Low-Gear Equivalent viscous damping coefficient	1.50×10^{-4} N-m/(rad/s)	
m_b	Mass of bar load	0.038 kg	
L_b	Length of bar load	0.1525 m	
m_d	Mass of disc load	0.04 kg	
r_d	Radius of disc load	0.05 m	
m_{max}	Maximum load mass	5 kg	
f	um input voltage frequency	50 Hz	
max	Maximum input current	1 A	
ω_{max}	Maximum motor speed	628.3 rad/s	

Table 3.1: Main SRV02 Specifications

Symbol	Description	Value
K_{gi}	Internal gearbox ratio	14
$K_{ge,low}$	Internal gearbox ratio (low-gear)	1
$K_{ge,high}$	Internal gearbox ratio (high-gear)	5
m_{24}	Mass of 24-tooth gear	0.005 kg
m_{72}	Mass of 72-tooth gear	0.030 kg
m_{120}	Mass of 120-tooth gear	0.083 kg
r_{24}	Radius of 24-tooth gear	6.35×10^{-3} m
r_{72}	Radius of 72-tooth gear	0.019 m
r_{120}	Radius of 120-tooth gear	0.032 m

Table 3.2: SRV02 Gearhead Specifications

Symbol	Description	Value	Variation
K_{pot}	Potentiometer sensitivity	35.2 deg/V	± 2 %
K_{enc}	Encoder sensitivity	4096 counts/rev	
K_{tach}	Tachometer sensitivity	1.50 V/k_{RPM}	± 2 %

Table 3.3: SRV02 Sensor Specifications

References

[1] Quanser Inc. *QUARC Installation Guide*, 2009.

[2] Quanser Inc. *Q2-USB User's Manual*, 2010.

[3] Quanser Inc. *SRV02 Assessment Workbook (Microsoft Excel file)*, 2010.

[4] Quanser Inc. *VoltPAQ User's Manual*, 2010.

[5] Quanser Inc. *QUARC User Manual*, 2011.

[6] Quanser Inc. SRV02 User Manual, 2011