YTÜ Mechanical Engineering Department

Lecture of Special Laboratory of Machine Theory, System Dynamics and Control Division

Coupled Tank 1 Level Control with using Feedforward PI Controller

Lab Report

Lab Date: **Number:**

Name Surname:

Lab Director: **Group/Sub-group:** ….. / ….

Lab Location: O Block-Otomatic Control Laboratory

Lab Name: Machine Theory 2

Subject: Coupled Tank 1 Level Control with using Feedforward PI Controller

Objectives

- Tune through pole placement the Pl-plus-feedforward controller for the actual water level in tank 1 of the Coupled-Tank system.
- Implement the Pl-plus-feedforward control loop for the actual Coupled-Tank's tank 1 level.
- Run the obtained Pl-plus-feedforward level controller and compare the actual response against the controller design specifications.
- Run the system's simulation simultaneously, at every sampling period, in order to compare the actual and simulated level responses.

1.1.1 Configuration #1 System Schematics

A schematic of the Coupled-Tank plant is represented in Figure 1, below. The Coupled-Tank system's nomenclature is provided in Appendix A. As illustrated in Figure 1, the positive direction of vertical level displacement is upwards, with the origin at the bottom of each tank (i.e. corresponding to an empty tank), as represented in Figure 1

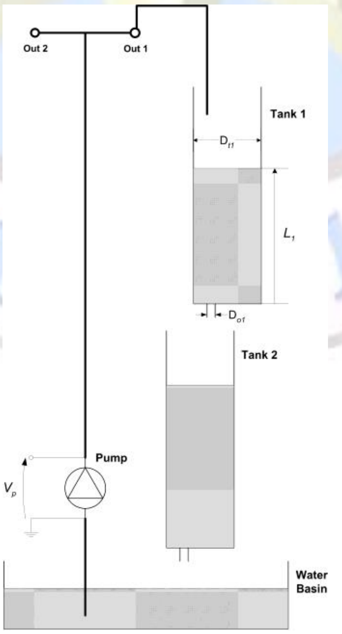


Figure 1: Schematic of Coupled Tank in Configuration #1.

Lab Questions

Using the notations and conventions described in Figure 2 derive the Equation Of Motion (EOM) characterizing the dynamics of tank 1. Is the tank 1 system's EOM linear?

Hint: The outflow rate from tank 1, F_{oi}, can be expressed by:

$$F_{o1} = A_{o1} v_{o1} \quad (2.12)$$

A-1 As a remark, the cross-section area of tank 1 outlet hole can be calculated by:

$$A_{o1} = \frac{1}{4}\pi D_{o1}^2 \quad \text{(Ans.2.1)}$$

Using Equation Ans.2.1, the outflow rate from tank 1 given in Equation 2.12 becomes:

$$F_{o1} = A_{o1}\sqrt{2gL_1} \quad \text{(Ans.2.2)}$$

A-2 Moreover, using the mass balance principle for tank 1, we obtained a first-order differential equation for L_1 in Equation 2.1. Substituting in Equation 2.1 F_{i1} and F_{o1} with their expressions given in Equation Ans.2.1 and Equation Ans.2.2, respectively, and rearranging results in the following equation of motion for the tank 1 system:

$$\frac{\partial L_1}{\partial t} = \frac{K_p V_p - A_{o1}\sqrt{2}\sqrt{gL_1}}{A_{t1}} \quad \text{(Ans.2.3)}$$

A-3 The EOM of tank 1 given in Equation Ans.2.3 is nonlinear.

The nominal pump voltage V_{p0} for the pump-tank 1 pair can be determined at the system's static equilibrium. By definition, static equilibrium at a nominal operating point (V_{p0}, L_{i0}) is characterized by the water in tank 1 being at a constant position level L_{i0} due to the constant inflow rate generated by V_{p0}. Express the static equilibrium voltage V_{p0} as a function of the system's desired equilibrium level L_{i0} and the pump flow constant Kp. Using the system's specifications given in the Coupled Tanks User Manual ([5]) and the desired design requirements, evaluate V_{p0} parametrically.

Outcome Solution

A-1 At equilibrium, all time derivative terms equate zero and Equation Ans.2.3 becomes:

$$K_p V_{p0} - A_{o1}\sqrt{2}\sqrt{gL_{10}} = 0 \quad \text{(Ans.2.4)}$$

$$V_{p0} =$$

Linearize tank 1 water level's EOM found in Question #1 about the quiescent operating point (V_{p0}, L_{10}).

A-1 Applying the Taylor's series approximation about (V_{p0},L_{10}), Equation Ans.2.3 can be linearized as represented below:

$$\frac{\partial L_1}{\partial t} = \frac{K_p V_{p0} - A_{o1}\sqrt{2}\sqrt{gL_{10}}}{A_{t1}} - \frac{1}{2}\frac{A_{o1}\sqrt{2}gL_{11}}{\sqrt{gL_{10}}A_{t1}} + \frac{K_p V_{p1}}{A_{t1}} \quad \text{(Ans.2.6)}$$

Substituting V_{p0} in Equation Ans.2.6 with its expression given in Equation Ans.2.5 results in the following linearized EOM for the tank 1 water level system:

$$\frac{\partial L_{11}}{\partial t} = -\frac{1}{2}\frac{A_{o1}\sqrt{2}gL_{11}}{\sqrt{gL_{10}}A_{t1}} + \frac{K_p V_{p1}}{A_{t1}} \qquad \text{(Ans.2.7)}$$

Determine from the previously obtained linear equation of motion, the system's open-loop transfer function in the Laplace domain as defined in Equation 2.5 and Equation 2.6. Express the open-loop transfer function DC gain, K_{dci}, and time constant, t1, as functions of L_{10} and the system parameters. What is the order and type of the system? Is it stable? Evaluate K_{dci} and t_1 according to system's specifications given in the Coupled Tanks User Manual ([5])

A-1 Applying the Laplace transform to Equation Ans.2.7 and rearranging yields:

$$K_{dc\_1} = \frac{K_p\sqrt{2}\sqrt{gL_{10}}}{A_{o1}g}, \qquad \tau_1 = \frac{A_{t1}\sqrt{2}\sqrt{gL_{10}}}{A_{o1}g} \qquad \text{(Ans.2.8)}$$

A-2 Such a system is stable since its unique pole (system of order one) is located on the left-hand-side of the s-plane. By not having any pole at the origin of the s-plane, Gi(s) is of type zero. Evaluating Equation Ans.2.8, accordingly to the system's parameters and the desired design requirements, gives:

$K_{dc\_1} =$

$\tau_1 =$

Specifications

In configuration #1, a control is designed to regulate the water level (or height) oftank#1 using the pump voltage. The control is based on a Proportional-Integral-Feedforward scheme (PI-FF). Given a ±1 cm square wave level setpoint (about the operating point), the level in tank 1 should satisfy the following design performance requirements:

1. Operating level in tank 1 at 15 cm: $\boldsymbol{L_{10} = 15}$ cm.
2. Percent overshoot less than 10%: $\boldsymbol{PO_1 < 11}$ %.
3. 2% settling time less than 5 seconds: $\boldsymbol{t_{s\_1} < 5.0}$ s.
4. No steady-state error: $\boldsymbol{e_{ss} = 0}$ cm.

Tank 1 Level Controller Design: Pole Placement

For zero steady-state error, tank 1 water level is controlled by means of a Proportional-plus-Integral (PI) closed-loop scheme with the addition of a feedforward action, as illustrated in Figure 2, below, the voltage feedforward action is characterized by:

$$V_{p\_ff} = K_{ff\_1}\sqrt{L_{r\_1}} \qquad (3.1)$$

and

$$V_p = V_{p1} + V_{p\_ff} \qquad (3.2)$$

As it can be seen in Figure 2, the feedforward action is necessary since the PI control system is designed to compensate for small variations (a.k.a. disturbances) from the linearized operating point (V_{p0}, L_{10}). In other words, while the feedforward action compensates for the water withdrawal (due to gravity) through tank 1 bottom outlet orifice, the PI controller compensates for dynamic disturbances.

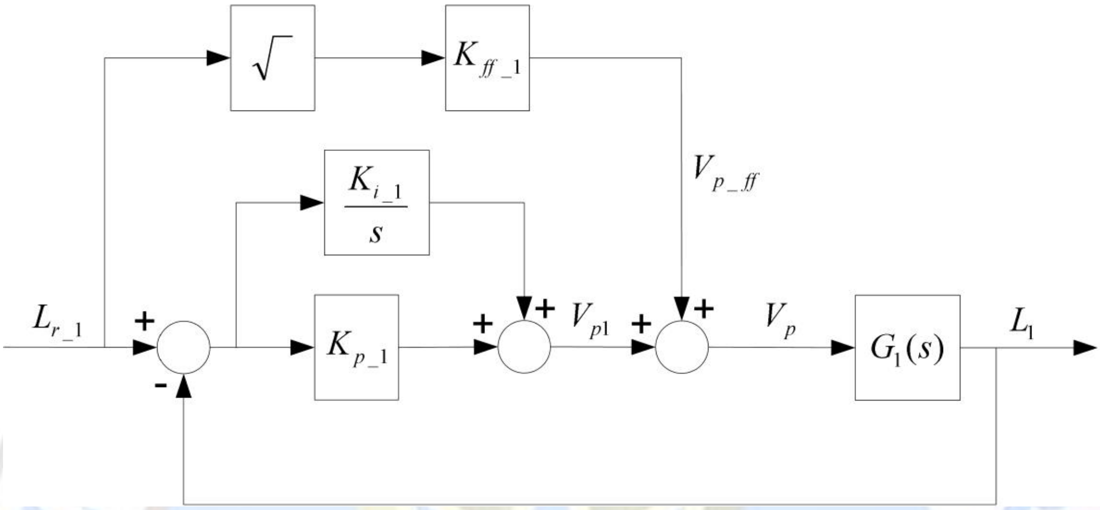


Figure 2: Tank 1 Water Level PI-plus-Feedforward Control Loop

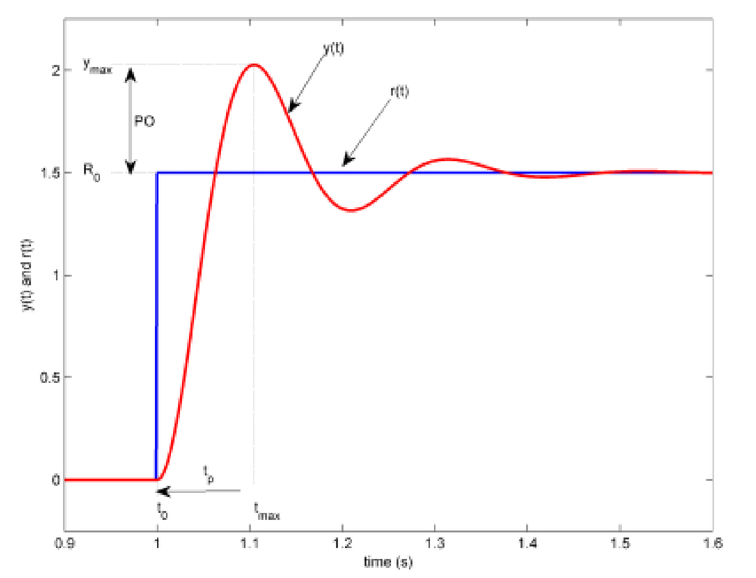


Figure 3: Standard second-order step response

The maximum value of the response is denoted by the variable $\boldsymbol{y_{max}}$ and it occurs at a time $\boldsymbol{t_{max}}$. For a response similar to Figure 3, the percent overshoot is found using

$$PO = \frac{100\ (y_{max} - R_0)}{R_0}$$

From the initial step time, t_0, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0$$

This is called the ***peak time*** of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100\, e^{\left(-\frac{\pi\,\zeta}{\sqrt{1-\zeta^2}}\right)}$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as

$$t_p = \frac{\pi}{\omega_n\,\sqrt{1-\zeta^2}}$$

Tank 1 level response 2% Settling Time can be expressed as follows:

$$t_s = \frac{4}{\zeta\omega}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

Lab Questions

Analyze tank 1 water level closed-loop system at the static equilibrium point (V_{p0}, L_{10}) and determine and evaluate the voltage feedforward gain, K_{ff_1}, as defined by Equation 3.1.

A-1 By definition, at the static equilibrium point (V_{p0}, L_{10}):

$$L_1 = L_{r\_1} = L_{10}, \qquad V_p = V_{p\_ff} = V_{p0} \qquad \text{(Ans.3.1)}$$

Using Equation Ans.2.5, the voltage feedforward gain results to be:

$$K_{ff\_1} = \frac{A_{o1}\sqrt{2g}}{K_p}$$

Evaluating Equation Ans.3.2 with the system's parameters given in the Coupled Tanks User Manual ([5]) leads to:

$$K_{ff\_1} =$$

Using tank 1 voltage-to-level transfer function $G_1(s)$ determined in Pre-Lab notes and the control scheme block diagram illustrated in Figure 2, derive the normalized characteristic equation of the water level closed-loop system.

Hint#1: The feedforward gain K_{ff_1} does not influence the system characteristic equation. Therefore, the feedforward action can be neglected for the purpose of determining the denominator of the closed-loop transfer function. Block diagram reduction can be carried out.

Hint#2: The system's normalized characteristic equation should be a function of the PI level controller gains, K_{p_1}, and K_{i_1}, and system's parameters, K_{dc_1} and r_1.

Outcome Solution

A-1 Neglecting the feedforward action and carrying out block diagram reduction using Equation 2.6 and Equation 3.4 one has

$$Y(s) = \frac{K_{dc\_1}}{\tau_1 s + 1}(K_{p1}(R(S) - Y(s)) + \frac{K_{i\_1}}{s}(R(S) - Y(s))) \quad \text{(Ans.3.4)}$$

which results in the following closed-loop transfer function

$$\frac{Y(s)}{R(s)} = \frac{K_{dc\_1}(K_{p\_1}s + K_{i\_1})}{\tau_1 s^2 + (1 + K_{dc\_1}K_{p\_1})s + K_{dc\_1}K_{i\_1}} \quad \text{(Ans.3.5)}$$

Re-arranging Equation Ans.3.5 results in tank 1 normalized characteristic polynomial:

$$s^2 + \frac{(1 + K_{dc\_1}K_{p\_1})s}{\tau_1} + \frac{K_{dc\_1}K_{i\_1}}{\tau_1} = 0 \quad \text{(Ans.3.6)}$$

By identifying the controller gains $\boldsymbol{K_{p\_1}}$ and K_{i_1}, fit the obtained characteristic equation to the second- order standard form expressed below:

$$s^2 + 2\zeta_1\omega_{n1}s + \omega_{n1}^2 = 0 \quad \text{(3.12)}$$

Determine K_{p_1} and $\boldsymbol{K_{i1}}$ as functions of the parameters $\boldsymbol{u_{n1}}$, C_1, $\boldsymbol{K_{dc\_1}}$, and $\boldsymbol{t_1}$ using Equation 3.5.

Solution

A-2 The system's desired characteristic equation is expressed by Equation Ans.3.6. Solving for the two unknowns $\boldsymbol{K_{p\_1}}$ and $\boldsymbol{K_{i\_1}}$ the set of two equations resulting from identifying the coefficients of Equation Ans.3.6 with those of Equation 3.12, the PI controller gains can be expressed as follows:

$$K_{p\_1} = \frac{2\zeta_1\omega_{n1}\tau_1 - 1}{K_{dc\_1}}, \qquad K_{i\_1} = \frac{\omega_{n1}^2\tau_1}{K_{dc\_1}} \quad \text{(Ans.3.7)}$$

Determine the numerical values for $\boldsymbol{K_{p\_1}}$ and $\boldsymbol{K_{i\_1}}$ in order for the tank 1 system to meet the closed-loop desired specifications, as previously stated.

A-1 The minimum damping ratio to meet the maximum overshoot requirement, $\boldsymbol{PO_1}$, can be obtained by solving Equation 3.9. The following relationship results:

$$\zeta_1 = \frac{ln(\frac{1}{100}PO_1)}{\sqrt{ln(\frac{1}{100}PO_1)^2 + \pi^2}}, \qquad K_{i\_1} = \frac{\omega_{n1}^2\tau_1}{K_{dc\_1}} \quad \text{(Ans.3.8)}$$

The system natural frequency, w_{n1}, can be calculated from Equation 3.11, as follows:

$$\omega_{n1} = \frac{4}{\zeta_1 t_{s\_1}}$$

A-2 Evaluating Equation Ans.3.8 and Equation Ans.3.9 according to the desired design requirements, then carrying out the numerical application of Equation Ans.3.7 leads to the following PI controller gains:

$K_{p\_1} =$

$K_{i\_1} =$

Tank 1 Level Control Implementation

The q_tanks_1 Simulink diagram shown in Figure 4 is used to perform the tank 1 level control exercises in this laboratory. The ***Coupled Tanks*** subsystem contains QUARC® blocks that interface with the pump and pressure sensors of the Coupled Tanks system.

Note that a first-order low-pass filter with a cut-off frequency of 2.5 Hz is added to the output signal of the tank 1 level pressure sensor. This filter is necessary to attenuate the high-frequency noise content of the level measurement.

Such a measurement noise is mostly created by the sensor's environment consisting of turbulent flow and circulating air bubbles. Although introducing a short delay in the signals, low-pass filtering allows for higher controller gains in the closed-loop system, and therefore for higher performance. Moreover, as a safety watchdog, the controller will stop if the water level in either tank 1 or tank 2 goes beyond 27 cm.

Experimental Setup

The ***q_tanks_1*** Simulink® diagram shown in Figure 4 will be used to run the PI+FF level control on the actual Coupled Tanks system.

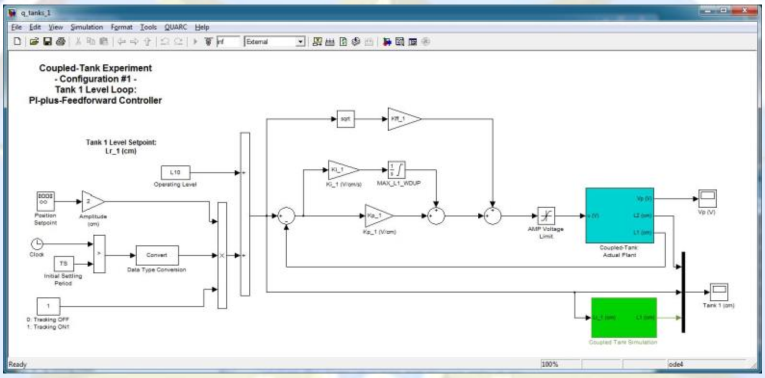


Figure 4: Simulink model used to run tank 1 level control on Coupled Tanks system.

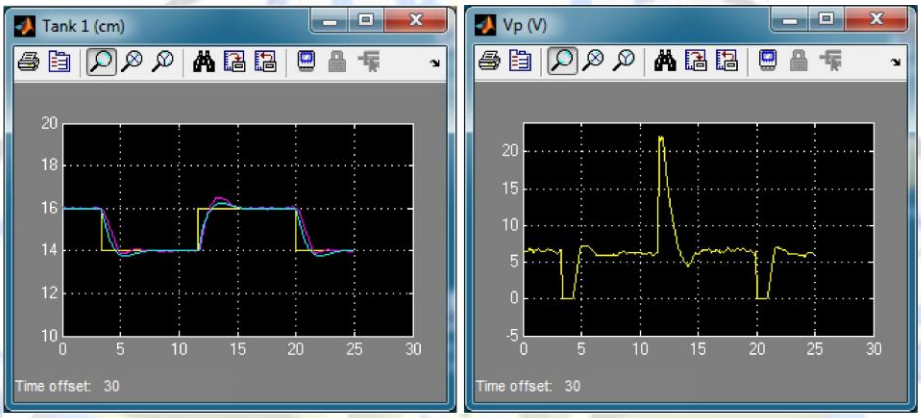
IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

Follow this procedure:

1. Enter the proportional, integral, and feed forward control gains found in Section 3.2 in Matlab®as K_{p1}, K_{i1}, K_{ff1}
2. To generate a step reference, go to the Signal Generator block and set it to the following:
   - Signal type = ***square***
   - Amplitude = 1
   - Frequency = 0.06 Hz
3. Set the ***Amplitude (cm)*** gain block to 1 to generate a square wave goes between ±1 cm.
4. Open the pump voltage ***Vp (V)*** and tank 1 level response ***Tank 1(cm)*** scopes.
5. By default, there should be anti-windup on the Integrator block (i.e., just use the default Integrator block).

6. In the Simulink diagram, go to QUARC | Build.
7. Click on QUARC | Start to run the controller. The pump should start running and filling up tank 1 to its operating level, $\boldsymbol{L}_{10}$. After a settling delay, the water level in tank 1 should begin tracking the ±1 cm square wave setpoint (about operating level Li_0).
8. II Generate a Matlab®figure showing the ***Implemented Tank 1 Control*** response and the input pump voltage

Expected figures have shown with Figure 5



(a) Tank 1 Level (b) Pump Voltage

Figure 5: Measured closed-loop tank 1 control response

Solution

If the procedure was followed properly, the control should have been ran on the Coupled Tanks system and the response similar to Figure 6 should have been obtained.

The closed-loop response is shown in Figure 6. You can generate this using the plot_tanks_***1***_rsp.m script.

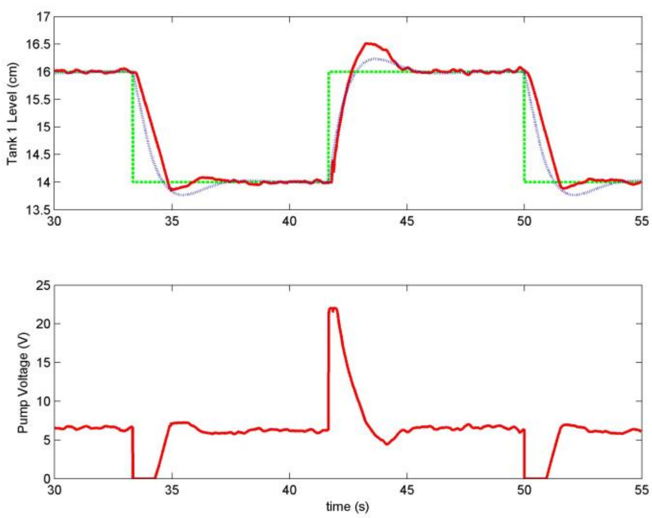


Figure 6: Closed-loop tank 1 level control response.

Outcome Solution

K-1 The settling time, percent overshoot, and steady-state error measured in the response shown in Figure 6are:

Description	Symbol	Value	Units
Pre Lab Questions			
Tank 1 Control Gains			
Feed Forward Control Gain	$K_{ff,1}$		VA/ cm
Proportional Control Gain	$K_{p,1}$		V/cm
Integral Control Gain	$K_{i,1}$		V/(cm-s)
Tank 1 Control Simulation			
Steady-state error	$e_{ss,\,1}$	0	cm
Settling time	$t_{s,\,1}$	1.9	s
Percent overshoot	PO_i	11.5	%
Tank 1 Control Implementation			
Steady-state error	$e_{ss,\,1}$		cm
Settling time	$t_{s,\,1}$		s
Percent overshoot	PO_i		%

Question: Why this difference are occurred? Explain it.